Econ 101A

Final Exam:
The Complete Reference Guide

All Topics · All Formulas · All Techniques

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How to Use This Guide

Part I (Sections 1–5): New final material — Weeks 9–12. Study these first; they carry the most weight.

Part II (Sections 6–12): Cumulative review formula sheets — one page per topic, everything you need for the "bring-it-all-together" questions.

Part III (Section 13–15): Execution toolkit — the Pattern Recognition Atlas, master formula sheet, and memory anchors. *Read this the night before.*

Contents

Part I — New Final Exam Material (Weeks 9–12)

1 General Equilibrium & the Edgeworth Box (Week 9)

Core Intuition

Partial equilibrium fixes the prices of all other markets. **General equilibrium** lets *all* prices adjust simultaneously until every market clears at once. In a pure exchange economy, no production occurs — agents only trade their endowments.

1.1 Definitions

A **Walrasian Equilibrium** $(x_A^*, x_B^*, p_1^*, p_2^*)$ requires:

1. *Individual optimality*: each consumer maximizes utility subject to their budget constraint, where **wealth** $= p_1\omega_1^i + p_2\omega_2^i$ (endowment valued at prices).
2. *Market clearing*: $x_j^{A*} + x_j^{B*} = \omega_j^A + \omega_j^B$ for all goods j.

Walras' Law: If $N-1$ markets clear, the Nth clears automatically. Use this to skip checking the second market.

1.2 The 4-Step Method to Find Equilibrium

Recipe: Pure Exchange GE

Step 1. Compute each consumer's *wealth* as a function of prices: $m_i(p) = p_1\omega_1^i + p_2\omega_2^i$.

Step 2. Solve each consumer's utility-max problem to find Marshallian demands $x_1^{i*}(p_1, p_2, m_i(p))$ and x_2^{i*}.

Step 3. Normalize: set $p_2 = 1$, let $p = p_1$. Impose market clearing for good 1:

$$x_1^{A*}(p) + x_1^{B*}(p) = \omega_1^A + \omega_1^B.$$

Solve for p^*.

Step 4. Substitute p^* back to find the equilibrium allocation. Verify Walras' Law on market 2 (should hold automatically).

1.3 Pareto Efficiency & the Contract Curve

An allocation is **Pareto efficient** if no reallocation can make one agent better off without making another worse off.

Efficiency Condition (tangency criterion)

At an interior Pareto-efficient allocation:

$$\boxed{MRS^A = MRS^B} \quad \Longleftrightarrow \quad \frac{MU_1^A}{MU_2^A} = \frac{MU_1^B}{MU_2^B}$$

The **Contract Curve** is the locus of all Pareto-efficient allocations inside the Edgeworth box. For Cobb-Douglas utilities $U_i = x_1^{a_i} x_2^{b_i}$, the contract curve is found by solving $MRS^A = MRS^B$ for the relationship between x_1 and x_2.

First Welfare Theorem: Every Walrasian equilibrium is Pareto efficient (the market "works").

Second Welfare Theorem: Any Pareto-efficient allocation can be achieved as a Walrasian equilibrium with the right initial endowments (redistribution + markets can get anywhere on the contract curve).

1.4 Worked Example

Classic Cobb-Douglas Symmetric Case

$U_A = x_1^A x_2^A$, $U_B = x_1^B x_2^B$. Endowments: $\omega^A = (2, 0)$, $\omega^B = (0, 2)$. Normalize $p_2 = 1$, $p = p_1$.
Wealth: $m_A = 2p$, $m_B = 2$.
Demands (Cobb-Douglas, $\alpha = \beta = 1/2$, so equal shares):

$$x_1^{A*} = \frac{m_A}{2p} = 1, \quad x_2^{A*} = \frac{m_A}{2} = p.$$

$$x_1^{B*} = \frac{m_B}{2p} = \frac{1}{p}, \quad x_2^{B*} = \frac{m_B}{2} = 1.$$

Market clearing: $x_1^{A*} + x_1^{B*} = 2 \Rightarrow 1 + \frac{1}{p} = 2 \Rightarrow \boxed{p^* = 1}$.
Allocation: $A = (1, 1)$, $B = (1, 1)$. Both consume equal amounts.
Efficiency check: $MRS^A = x_2^A / x_1^A = 1 = MRS^B$, and $p^*/p_2 = 1$. ✓

Common Traps

- **Endowment income**: wealth is $p\omega_x + \omega_y$, not a fixed m. It changes with p.
- **Normalize correctly**: set $p_2 = 1$. Only relative prices matter.
- **Leontief consumer**: use the kink condition, not Lagrangian.
- **Market clearing vs. excess demand**: set $\sum_i x_j^{i*} = \sum_i \omega_j^i$.

2 Game Theory Primer (Week 10)

Core Definitions

- **Normal-form game**: players $i \in \{1, \ldots, I\}$, strategy sets S_i, payoffs $U_i(s_i, s_{-i})$.
- **Nash Equilibrium**: a strategy profile $(s_1^*, \ldots, s_I^*)$ such that no player can improve their payoff by deviating unilaterally.
- **Dominant strategy**: best response regardless of what others do.
- **Dominant strategy equilibrium $\subset$ Nash Equilibrium**.

2.1 Finding Nash Equilibria

Method 1 (Discrete games — best-response check): For each possible strategy profile, check whether either player wants to deviate. Mark best responses with ✓. A cell where all players have ✓ is a Nash equilibrium.

Method 2 (Continuous strategy spaces): Differentiate each player's payoff w.r.t. their own action, holding others' actions fixed. Solve the system of FOCs simultaneously.

2.2 Prisoner's Dilemma & Repeated Games

Prisoner's Dilemma

	Silent	**Betray**
Silent	$(-1, -1)$	$(-10, 0)$
Betray	$(0, -10)$	$(-5, -5)$

"Betray" is a **dominant strategy** for both $\Rightarrow$ unique Nash equilibrium: (Betray, Betray). This is *Pareto dominated* by (Silent, Silent). That is the tragedy of the PD.

Repeated Games: Trigger Strategy

With **finitely** repeated PD (T finite, common knowledge): backward induction unravels cooperation. Play (Betray, Betray) in every period.
With **infinitely** repeated PD (or unknown end date): cooperation can be sustained. **Trigger strategy**: cooperate as long as opponent cooperates; defect forever after one defection. Cooperation sustainable if the discount factor δ is high enough (players are patient).

3 Oligopoly: Cournot, Bertrand, Stackelberg (Weeks 10–11)

3.1 Cournot (Quantity Competition, Simultaneous)

Memory Anchor — "Two Wolves Sharing a Carcass"

Each firm chooses quantity *assuming the other's quantity is fixed*. Neither firm wants to deviate given the other's choice. The carcass shrinks as they both eat — so price falls with total output.

Cournot Recipe (Linear Demand)

Setup: $P = a - b(q_1 + q_2)$, marginal cost c (same for both).

Step 1. Write each firm's profit:

$$\pi_i = [a - b(q_i + q_{-i}) - c]q_i.$$

Step 2. FOC in q_i (hold q_{-i} fixed):

$$a - 2bq_i - bq_{-i} - c = 0 \quad \Rightarrow \quad q_i(q_{-i}) = \frac{a - c - bq_{-i}}{2b}.$$

Step 3. Symmetric NE ($q_1^* = q_2^* = q^*$):

$$q^* = \frac{a-c}{3b}, \qquad Q^* = \frac{2(a-c)}{3b}, \qquad P^* = \frac{a+2c}{3}.$$

Profit per firm: $\pi^* = (a-c)^2/(9b)$.

n-firm generalization: $q_i^* = \dfrac{a-c}{(n+1)b}$; as $n \to \infty$, $P^* \to c$ (competitive).

Critical rule: *Never substitute $q_i = q_{-i}$ before taking the FOC.* Impose symmetry only *after* the FOC is derived.

3.2 Bertrand (Price Competition, Simultaneous)

Bertrand Paradox

Two (or more) identical firms, each with marginal cost c, competing on **price**.
Unique Nash Equilibrium: $p_1^* = p_2^* = c$ (price = marginal cost).
Why: if $p_{-i} > c$, firm i can undercut by ϵ and capture the whole market. This undercutting continues until neither firm can profitably reduce price further.
Implication: even two firms produce the competitive outcome. Zero profits. "Bertrand gives PC from 2 firms; Cournot needs $n \to \infty$."

3.3 Stackelberg (Leader-Follower, Sequential)

Stackelberg Recipe (Backward Induction)

Step 1. Solve follower's problem (Firm 2, takes q_1 as given):

$$q_2^*(q_1) = \frac{a - c - bq_1}{2b}.$$

Step 2. Leader internalizes follower's response — substitute $q_2^*(q_1)$ into Firm 1's profit:

$$\pi_1 = \left[a - bq_1 - b \cdot \frac{a - c - bq_1}{2b} - c\right] q_1.$$

Step 3. FOC for Firm 1:

$$q_1^* = \frac{a-c}{2b}, \qquad q_2^* = \frac{a-c}{4b}.$$

Leader produces *more* than in Cournot ($q_1^S > q_1^{CN}$). Follower produces *less*. **First-mover advantage**: $\pi_1^S > \pi_i^{CN} > \pi_2^S$.

3.4 The Comparison Table (Memorize This)

Market Structure Comparison (Linear Demand, Symmetric Firms)

Structure	q_i^*	Q^*	P^*	π_i^*
Perfect Comp. ($n \to \infty$)	$\to 0$	$(a-c)/b$	c	0
Cournot duopoly	$(a-c)/(3b)$	$2(a-c)/(3b)$	$(a+2c)/3$	$(a-c)^2/(9b)$
Stackelberg leader	$(a-c)/(2b)$	$3(a-c)/(4b)$	$(a+3c)/4$	$(a-c)^2/(8b)$
Stackelberg follower	$(a-c)/(4b)$			$(a-c)^2/(16b)$
Monopoly	$(a-c)/(2b)$	$(a-c)/(2b)$	$(a+c)/2$	$(a-c)^2/(4b)$

Ordering of output: PC > Stackelberg > Cournot > Monopoly.
Ordering of price: PC < Stackelberg < Cournot < Monopoly.

Common Traps

- Cournot: differentiate FIRST, then impose symmetry.
- Stackelberg: firm 1's FOC changes (compared to Cournot) because it accounts for $\partial q_2^*/\partial q_1 = -1/2$.
- Bertrand with capacity constraints or differentiated products: outcome is no longer $p = c$.

4 Externalities & Pigouvian Correction (Week 12)

Memory Anchor — "The Phantom Cost"

An externality is a cost (or benefit) the decision-maker doesn't see. The Pigouvian tax *makes the phantom visible*: it forces the firm to internalize the social cost.

4.1 Definition & Types

Externality: an agent's action affects another's payoff through a channel *not mediated by a price*.

- **Negative production externality**: steel firm pollutes river, raising fishery's costs.
- **Positive production externality**: orchard's bees benefit neighboring farm (free pollination).
- **Consumption externality**: neighbor's loud music (negative); neighbor's flower garden (positive).

Key insight: externalities = *missing markets*. If pollution had a price, firms would internalize it. This is why the First Welfare Theorem *fails* with externalities.

4.2 The Negative Externality: Private vs. Social Optimum

Private vs. Social Optimum

Simple one-firm setup: firm produces q at private cost $C(q)$, sells at price p, generates external damage $D(q)$ on others (e.g., pollution).
Private FOC (firm ignores externality):

$$p = C'(q^P).$$

Social planner's FOC (maximizes total surplus $= \pi - D(q)$):

$$p = C'(q^S) + D'(q^S).$$

Since $D'(q) > 0$: $\quad C'(q^S) < C'(q^P) \quad \Rightarrow \quad q^S < q^P$. The firm **overproduces** relative to the social optimum.

Pigouvian Tax

A per-unit tax t^* on output shifts the firm's FOC to:

$$p - t^* = C'(q).$$

Set $t^* = D'(q^S)$ (marginal external damage *evaluated at the social optimum*). Then the firm privately chooses q^S. The tax *makes the phantom visible*.
For a positive externality: use a Pigouvian *subsidy* s^* = marginal external benefit at q^S.

4.3 Worked Example

Numerical Example

Factory: private cost $C(q) = q^2/2$, price $p = 10$, damage $D(q) = q^2$.

Private: $p = C'(q^P) \Rightarrow 10 = q^P \Rightarrow q^P = 10$.
Social: $p = C'(q^S) + D'(q^S) \Rightarrow 10 = q^S + 2q^S \Rightarrow q^S = 10/3$.
Tax: $t^* = D'(q^S) = 2(10/3) = 20/3 \approx 6.67$.

DWL of laissez-faire:

$$\text{DWL} = \int_{q^S}^{q^P} [MSC(q) - p]\, dq = \int_{10/3}^{10} (3q - 10)\, dq = \frac{200}{3} \approx 66.67.$$

4.4 Steel-Fishery Model (Gaubert's Favorite)

Two-Firm Production Externality

Firm S (steel) has cost $c_S(s, x)$ where x = pollution ($\partial c_S/\partial x \leq 0$: more pollution lowers S's cost).
Firm F (fishery) has cost $c_F(f, x)$ where $\partial c_F/\partial x > 0$: pollution raises F's cost.
Competitive outcome: S chooses x to max own profit; FOC: $\partial c_S/\partial x = 0$. Pollutes until no more private benefit. F's losses are ignored.
Social planner: FOC for pollution:

$$\frac{\partial c_S}{\partial x} + \frac{\partial c_F}{\partial x} = 0.$$

The planner sets x where S's marginal gain from pollution equals F's marginal damage.
Pigouvian tax: charge S a tax equal to F's marginal damage at x^S, so S internalizes F's cost in its private FOC.

4.5 Coase Theorem (Conceptual)

Coase Theorem

With **zero transaction costs** and **well-defined property rights**, private bargaining achieves the social optimum regardless of who holds the property right.

- If S has the right to pollute: F *pays* S to reduce pollution down to x^S.
- If F has the right to clean river: S *pays* F to allow pollution up to x^S.
- Same outcome, different distribution of surplus.

Pigou vs. Coase: complementary, not contradictory. Same efficient target, different instrument.
Real-world failure: transaction costs are rarely zero (many parties, information problems).

5 Asymmetric Information & Adverse Selection (Week 12)

This is new material not in prior exams. Expect at least a conceptual question.

Core Intuition

Adverse selection arises when one side of the market has private information about quality, and the *selection of who trades* is endogenous to price. At a low price, only low-quality sellers participate $\Rightarrow$ buyers' beliefs are rational $\Rightarrow$ prices are low $\Rightarrow$ good sellers exit $\Rightarrow$ **market unravels**.

5.1 Akerlof's Lemons Model

Akerlof Setup (Used Cars)

Sellers: own cars of quality $\theta \in \{\text{Lemon (L)}, \text{Good (G)}\}$.

- Lemon: worth \$5,000 to seller; \$7,500 to buyer.
- Good car: worth \$10,000 to seller; \$15,000 to buyer.
- Fraction q of cars are good; fraction $1 - q$ are lemons.

Buyers: cannot observe car quality. **Sellers**: know their own car's quality.
Key: trade is efficient for BOTH types (buyer values exceed seller values). Without information problems, all cars should trade.

Equilibrium Analysis

Buyers' WTP: depends on their belief $\mu = \Pr(\text{good car offered at price } P)$:

$$P^*(\mu) = \mu \cdot 15{,}000 + (1 - \mu) \cdot 7{,}500 = 7{,}500 + \mu \cdot 7{,}500.$$

Seller participation:

- Lemon seller always sells: $P^*(\mu) \geq 5{,}000$ for any μ.
- Good seller only sells if: $P^*(\mu) \geq 10{,}000 \Rightarrow \mu \geq 1/3$.

Two equilibria:

1. **Pooling** (both types sell): requires $q \geq 1/3$, then $\mu = q$, $P^* = 7{,}500 + 7{,}500q$.
2. **Lemons equilibrium** (only lemons sell): $\mu = 0 \Rightarrow P^* = 7{,}500$. Good-car owners refuse to sell at this price. *Market for good cars collapses.*

When $q < 1/3$: only the lemons equilibrium exists. Efficient trade is destroyed by information asymmetry alone.

Key Insight & Real-World Applications

Mechanism: the price determines who is willing to sell; who sells determines the quality mix; the quality mix determines the price. This circular dependence is **adverse selection**.
Real-world analogues:

- **Health insurance**: only sick people buy $\Rightarrow$ premiums rise $\Rightarrow$ healthy drop out $\Rightarrow$ unraveling.
- **Credit markets**: borrowers who expect to default are most eager to borrow.
- **Labor markets**: workers who accept flat offers may signal low outside options.

Remedies: warranties/signals (costly for lemons to mimic); inspections/certifications; mandates (force all types to participate and prevent unraveling).

What to Know for the Exam

- Distinguish adverse selection (hidden information, pre-contractual) from moral hazard (hidden action, post-contractual).
- Know the Akerlof mechanism: endogenous selection $\Rightarrow$ market unraveling.
- Be able to solve for equilibrium price(s) given buyer WTP and seller participation conditions.
- Know why the First Welfare Theorem fails here: markets are incomplete.

Part II — Cumulative Review: High-Yield Formula Sheets

6 Consumer Theory (Weeks 1–4)

6.1 Marshallian Demand

The Three Utility Types You Must Know Cold

(1) Cobb-Douglas $U = x^a y^b$:

$$x^* = \frac{a}{a+b} \cdot \frac{m}{p_x}, \qquad y^* = \frac{b}{a+b} \cdot \frac{m}{p_y}.$$

Anchor: "Fixed-Share Wallet" — spend a fixed fraction of income on each good.
(2) Quasilinear $U = v(x) + y$: FOC gives $v'(x^*) = p_x/p_y$. Solve for x^* — it is *independent of income* m. Then $y^* = (m - p_x x^*)/p_y$ (can be negative $\Rightarrow$ corner solution at $y = 0$).
(3) Perfect Complements $U = \min(\alpha x, \beta y)$: Kink condition: $\alpha x = \beta y \Rightarrow y = (\alpha/\beta)x$. Sub into budget: $x^* = m/(p_x + (\alpha/\beta)p_y)$.

6.2 Duality: Hicksian Demand & Expenditure Function

Duality Relationships

$$\begin{aligned}
\text{Indirect utility: } & v(p, m) = \text{max utility given } (p, m). \\
\text{Expenditure fn: } & e(p, u) = \text{min expenditure to achieve utility } u. \\
\text{Hicksian demand: } & h_x(p, u) = \partial e/\partial p_x \quad \text{(Shephard's Lemma)}. \\
\text{Roy's Identity: } & x^*(p, m) = -(\partial v/\partial p_x)/(\partial v/\partial m).
\end{aligned}$$

Inversion: $v(p, e(p, u)) = u$ and $e(p, v(p, m)) = m$.

6.3 Slutsky Equation

Slutsky Decomposition

$$\underbrace{\frac{\partial x^*}{\partial p_x}}_{\text{total effect}} = \underbrace{\frac{\partial h_x}{\partial p_x}}_{\substack{\text{substitution} \\ \text{effect } (\leq 0)}} - \underbrace{x^* \cdot \frac{\partial x^*}{\partial m}}_{\text{income effect}}.$$

Normal good: income effect reinforces substitution $\Rightarrow$ demand slopes down.
Giffen good: income effect $>$ substitution effect (in magnitude) $\Rightarrow$ demand slopes *up*.

7 Intertemporal Choice & Labor Supply (Week 5)

Two-Period Consumption Model

Budget constraint (present value):

$$c_0 + \frac{c_1}{1+r} = y_0 + \frac{y_1}{1+r} \equiv W.$$

Euler Equation (FOC of intertemporal utility $U(c_0) + \beta U(c_1)$):

$$U'(c_0) = \beta(1+r)U'(c_1).$$

If $\beta(1+r) = 1$: perfect consumption smoothing ($c_0^* = c_1^*$). If $\beta(1+r) > 1$ (patient): $c_1^* > c_0^*$ (save).

Labor-Leisure Tradeoff

Consumer has time endowment T. Labor $L = T - \ell$, wage w, non-labor income m.
Utility $U(c, \ell)$, budget: $c = w(T - \ell) + m \Rightarrow c + w\ell = wT + m$.
Optimal condition: $MRS_{c,\ell} = MU_\ell / MU_c = w$ (value of leisure = wage).
Wage increase has two effects: substitution effect (leisure more expensive ⇒ work more) and income effect (richer ⇒ buy more leisure ⇒ work less). Labor supply can be backward-bending.

8 Producer Theory & Cost Functions (Week 6)

Cost Minimization (Dual of Consumer Expenditure)

$$\min_{K,L} wL + rK \quad \text{s.t.} \quad f(K, L) \geq y.$$

Lagrangian FOC: $\dfrac{f_L}{f_K} = \dfrac{w}{r} \quad \Leftarrow$ MRTS = input price ratio.
Shephard's Lemma: $\partial c(w, r, y)/\partial w = L^*(w, r, y)$ (conditional factor demand).
Returns to scale check: if $f(\lambda K, \lambda L) = \lambda^\nu f(K, L)$:

- $\nu > 1$: IRS ⇒ $c(w, r, y)$ is concave in y.
- $\nu = 1$: CRS ⇒ $c(w, r, y) = c(w, r) \cdot y$ (constant MC).
- $\nu < 1$: DRS ⇒ $c(w, r, y)$ is convex in y.

Profit Maximization

$$\max_y \; py - c(w, r, y).$$

FOC: $p = MC(y)$. **SOC**: $MC'(y) \geq 0$ (upward-sloping MC).
For multi-input $f(K, L)$, the Hessian of $-\pi$ must be positive semi-definite (NSD of profit Hessian):

$$\begin{vmatrix} \pi_{LL} & \pi_{LK} \\ \pi_{KL} & \pi_{KK} \end{vmatrix} > 0.$$

9 Competitive Equilibrium & IFT (Week 7)

Market Clearing & The IFT Toolkit

Equilibrium condition: $D(p^*, \alpha) = S(p^*, \alpha)$, where α is an exogenous parameter (e.g., tax).
Define $F(p, \alpha) \equiv D(p, \alpha) - S(p, \alpha) = 0$.
Implicit Function Theorem gives:

$$\frac{\partial p^*}{\partial \alpha} = -\frac{F_\alpha}{F_p} = -\frac{\partial D/\partial \alpha - \partial S/\partial \alpha}{\partial D/\partial p - \partial S/\partial p}.$$

Sign it: $\partial D/\partial p < 0$, $\partial S/\partial p > 0 \Rightarrow$ denominator < 0. Evaluate numerator to determine direction.
Tax incidence: for a per-unit tax t on supply, $S(p, t) = S^0(p - t)$. Then $\partial p^*/\partial t = S'/(D' - S')$. The burden falls more on the *less elastic* side.

10 Monopoly & Price Discrimination (Weeks 8–9)

Monopoly Profit Maximization

FOC: $MR(q) = MC(q)$. Recall: if $P(q) = a - bq$, then $MR(q) = a - 2bq$.
Markup rule (Lerner index): $\dfrac{P - MC}{P} = \dfrac{1}{|\varepsilon|}$, where $\varepsilon =$ price elasticity of demand.
DWL $= \int_{Q_m}^{Q_c} [P(q) - MC(q)]\, dq =$ triangle between demand and MC from Q_m to Q_c.

Third-Degree Price Discrimination

Monopolist sells in two segmented markets (markets 1 and 2). Profit:

$$\max_{q_1, q_2} P_1(q_1)q_1 + P_2(q_2)q_2 - C(q_1 + q_2).$$

FOC: $MR_1 = MR_2 = MC$.
Inverse elasticity rule: $P_i \left(1 - \dfrac{1}{|\varepsilon_i|}\right) = MC$. Higher price in the less elastic market ($|\varepsilon_1| < |\varepsilon_2| \Rightarrow P_1 > P_2$).

11 Risk & Expected Utility

Memory Anchor — "The Concave Hill"

Risk aversion = concave utility. Expected payoff is on the chord; expected utility is *below* the chord (under the curve). The gap is the risk premium. Jensen's Inequality: $E[U(w)] \leq U(E[w])$.

Risk Aversion Measures

Arrow-Pratt ARA: $A(w) = -U''(w)/U'(w)$.

Function	ARA	Property
$U = \sqrt{w}$	$1/(2w)$	DARA
$U = \ln w$	$1/w$	DARA
$U = -e^{-\gamma w}$	γ (constant)	CARA
$U = w^{1-\rho}/(1-\rho)$	ρ/w	CRRA (DARA)

Certainty Equivalent (CE): $U(CE) = E[U(\tilde{w})]$. **Risk premium**: $\pi = E[\tilde{w}] - CE \geq 0$.
CARA + Normal: if $\tilde{w} \sim N(\mu, \sigma^2)$ and $U = -e^{-\gamma w}$:

$$\boxed{CE = \mu - \frac{\gamma}{2}\sigma^2.}$$

Memorize this. Berkeley loves CARA + Normal.

12 Robinson Crusoe: The Synthesis Problem

Memory Anchor — "The Single-Brain Economy"

Robinson is simultaneously consumer, firm, and market. Prices are bookkeeping. The First Welfare Theorem says the market solution = the planner's solution. It's a no-schizophrenia theorem.

Robinson Crusoe Recipe

Setup: $U(c, \ell)$, time endowment T, production $c = f(L)$ where $L = T - \ell$.
Step A (Planner's problem): $\max_\ell U(f(T-\ell), \ell)$. First-order condition pins ℓ^*, L^*, c^*.
Step B (Decentralized equilibrium):

1. *Firm*: $\max_L f(L) - wL$. FOC: $f'(L^d) = w \Rightarrow L^d(w)$. Profit $\pi(w) = f(L^d) - wL^d$.
2. *Consumer*: income $= wT + \pi(w)$. Maximize $U(c, \ell)$ s.t. $c + w\ell = wT + \pi$.
3. *Market clearing*: $L^d(w) = T - \ell^*(w)$. Solve for w^*.

Step C: verify decentralized = planner's allocation. This is the **First Welfare Theorem**.
CRS edge case: if $f(L) = L$, then profit $= (1-w)L$. Need $w^* = 1$ for any finite equilibrium. Zero profit.

Part III — Exam Execution Toolkit

13 Pattern Recognition Atlas

Trigger ⇒ Tool: One-Glance Lookup

When you see...	Reach for...
$U = x^a y^b$	Income-share rule: $x^* = \frac{a}{a+b}\frac{m}{p_x}$.
$U = \min(\alpha x, \beta y)$ (Leontief)	Kink: $\alpha x = \beta y$, sub into budget.
$U = v(x) + y$ (quasilinear)	$v'(x^*) = p_x/p_y$; no income effect on x.
"Find Hicksian" / "expenditure fn"	Dual problem + Shephard's Lemma.
"Decompose total effect of price"	Slutsky equation.
"Prove direction without solving"	Implicit Function Theorem on FOC.
"Verify it's a maximum" (2 variables)	Hessian: $\lvert H_1\rvert < 0$, $\lvert H_2\rvert > 0$.
"Per-unit tax" ⇒ equilibrium shift	IFT on $D(p) = S(p - t)$.
"Two markets" / segmented monopoly	Inverse elasticity rule: $MR_1 = MR_2 = MC$.
"Risky payoff" / "insurance"	Expected utility, CE, Arrow-Pratt.
CARA + Normal payoff	$CE = E[w] - \frac{\gamma}{2}\text{Var}(w)$.
"Externality" / "pollution"	Pigouvian $t^* = MED$ at q^S.
"Two firms, same good, simultaneous q"	Cournot: best-response lines, symmetric NE.
"Two firms, same good, simultaneous p"	Bertrand: $p_1^* = p_2^* = c$.
"Firm 1 moves first"	Stackelberg: backward induction.
"Consumer + firm together" / island	Robinson Crusoe; First Welfare Theorem.
"Pure exchange" / Edgeworth box	Walras' Law; $MRS^A = MRS^B$ for efficiency.
"Hidden quality" / "used cars"	Akerlof adverse selection; lemons equilibrium.

14 Master Formula Cheat Sheet

Consumer Theory

CD demand: $x^* = \frac{a}{a+b}\frac{m}{p_x}$
Slutsky: $\partial x^*/\partial p_x = \partial h_x/\partial p_x - x^* \cdot \partial x^*/\partial m$
Shephard: $h_x = \partial e/\partial p_x$
Roy: $x^* = -v_{p_x}/v_m$
Euler: $U'(c_0) = \beta(1 + r)U'(c_1)$

Producer Theory

MRTS = price ratio: $f_L/f_K = w/r$
SOC: $f_{LL} < 0$, $\lvert H\rvert = f_{LL}f_{KK} - f_{LK}^2 > 0$
Profit max: $p = MC$
Lerner: $(P - MC)/P = 1/\lvert\varepsilon\rvert$

IFT on Market Clearing

$F(p, \alpha) = D - S = 0$:
$\partial p^*/\partial \alpha = -F_\alpha / F_p$

Oligopoly

Cournot NE: $q^* = \frac{a-c}{3b}$, $P^* = \frac{a+2c}{3}$
Bertrand NE: $p^* = c$ (zero profit)

Stackelberg: $q_1^* = \frac{a-c}{2b}$, $q_2^* = \frac{a-c}{4b}$
n-firm: $q_i^* = \frac{a-c}{(n+1)b}$

Risk

ARA: $A(w) = -U''/U'$
CE definition: $U(CE) = E[U(\tilde{w})]$
CARA-Normal: $CE = \mu - \frac{\gamma}{2}\sigma^2$
Risk premium: $\pi = E[\tilde{w}] - CE$

General Equilibrium

Walras' Law: if $N-1$ markets clear, Nth clears
Efficiency: $MRS^A = MRS^B$
Wealth: $m_i = p_1\omega_1^i + p_2\omega_2^i$
1st Welfare Thm: CE ⇒ Pareto efficient

Externalities

Private FOC: $p = C'(q^P)$
Social FOC: $p = C'(q^S) + D'(q^S)$
Pigouvian: $t^* = D'(q^S)$
DWL: $\int_{q^S}^{q^P} [MSC - MB]\, dq$

Adverse Selection

Buyer WTP: $P(\mu) = 7500 + \mu \cdot 7500$
Good seller participates: $\mu \geq 1/3$
Lemons eq: $\mu = 0$, only lemons trade

15 The Master Memory Anchor Stack

13 Anchors — One Sentence Each

1. **Fixed-Share Wallet** (Cobb-Douglas): fixed fraction of income on each good.
2. **Necessity Bucket** (quasilinear): income effect zero; extra wealth goes to the other good.
3. **Patience Tug-of-War** (Euler): $\beta R > 1 \Rightarrow$ save; $\beta R < 1 \Rightarrow$ borrow.
4. **Rubber Band** (labor supply): wages pull labor up; income effect pulls leisure back.
5. **Mirror Dimension** (cost min): producer theory is consumer theory reflected.
6. **Bowl Holding Water** (profit-max SOC): concave production ⇒ unique max.
7. **Inelastic Sucker** (tax incidence): stiffer side of the market absorbs the tax.
8. **Bicycle Lock** (price discrimination): turn each market dial to $MR_i = MC$.
9. **Auctioneer's Coin** (general equilibrium): one relative price clears all markets.
10. **Concave Hill** (risk aversion): chord above curve; vertical gap = risk premium.
11. **Phantom Cost** (externality): Pigou makes the invisible cost visible.
12. **Two Wolves** (Cournot): each takes the other's bite as fixed; NE = both committed.
13. **Single-Brain Economy** (Robinson Crusoe): planner = market; no schizophrenia.
14. **Endogenous Selection** (adverse selection): price determines who sells; who sells determines quality; quality determines price.

Top 5 Exam-Day Mistakes to Avoid

1. **Cournot**: taking FOC with symmetry ($q_i = q_{-i}$) already imposed. Always differentiate first.
2. **GE wealth**: treating endowment value as fixed. Wealth changes with prices: $m_i = p\omega_x^i + \omega_y^i$.
3. **IFT**: forgetting to check the sign of the denominator separately from the numerator.

4. **Pigouvian tax**: setting $t^* = D'(q^P)$ (private quantity) instead of $D'(q^S)$ (social optimum).
5. **Leontief demands**: trying to use Lagrangian instead of the kink condition.

Good luck, Vedant.

You have everything you need. Yuuuuuur.