
Comprehensive Microeconomic Theory

From Principles to Advanced Analysis

Econ 100A · Econ 101A · Graduate Extensions

Vedant | UC Berkeley | Spring 2026

How to Use This Document

Each topic is presented in three tiers:

- Tier 1** Principles-level intuition (Econ 100A / Campbell)
 - Tier 2** Rigorous intermediate analysis (Econ 101A / Gaubert)
 - Tier 3** Graduate treatment (Varian, duality, envelope theorem)
- Exam focus:** Final exam is Tier 2; Tier 3 gives the deep “why.”

Contents

| | | |
|------------|--|-----------|
| I | Foundations: Rationality, Preferences & Utility | 2 |
| 1 | Rationality & Preferences | 2 |
| 1.1 | Convexity and Its Economic Meaning | 2 |
| 2 | Utility Functions & Indifference Curves | 3 |
| 2.1 | Standard Functional Forms | 3 |
| II | Consumer Theory | 3 |
| 3 | Marshallian Demand: The Utility Maximization Problem | 4 |
| 4 | Duality: The Expenditure Function & Hicksian Demand | 5 |
| 5 | The Slutsky Equation: Decomposing Price Effects | 6 |
| 6 | Elasticity, Consumer Surplus & Tax Incidence | 7 |
| 7 | Intertemporal Choice & The Euler Equation | 8 |
| 8 | Labor Supply | 9 |
| 9 | Risk & Expected Utility Theory | 9 |
| III | Producer Theory | 10 |
| 10 | Production, Cost Minimization & Conditional Factor Demands | 11 |
| 11 | Profit Maximization & Competitive Supply | 12 |
| 12 | Long-Run Competitive Equilibrium | 12 |
| IV | Market Equilibrium & Comparative Statics | 12 |
| 13 | Competitive Equilibrium & the IFT | 13 |
| V | Market Power | 13 |
| 14 | Monopoly & Price Discrimination | 13 |
| 15 | Oligopoly & Game Theory | 14 |

| | | |
|-------------|---|-----------|
| VI | General Equilibrium & Welfare | 15 |
| 16 | Pure Exchange Economy & Walrasian Equilibrium | 15 |
| 17 | Robinson Crusoe Economy | 17 |
| VII | Market Failures | 17 |
| 18 | Externalities & Pigouvian Taxation | 17 |
| 19 | Public Goods & the Samuelson Condition | 18 |
| VIII | Information, Incentives & Behavioral Economics | 18 |
| 20 | Asymmetric Information: Adverse Selection | 18 |
| 21 | Behavioral Economics | 19 |
| IX | Appendix A: Master Formula Reference | 20 |
| A | Master Formula Cheat Sheet | 20 |
| B | Pattern Recognition Atlas | 21 |

Part I

Foundations: Rationality, Preferences & Utility

1. Rationality & Preferences

TIER 1 — Principles (Econ 100A): What Are Preferences?

A preference relation \succsim on a set X reads “is at least as good as.” We say $x \succsim y$ if the consumer weakly prefers bundle x to y . From \succsim we derive:

- **Strict preference** \succ : $x \succ y$ but not $y \succ x$
- **Indifference** \sim : $x \succsim y$ and $y \succsim x$

Preferences are the primitive of consumer theory — they exist before any utility function.

TIER 2 — Intermediate (Econ 101A): The Four Axioms

For preferences to be rational and representable:

1. **Completeness**: $\forall x, y \in X: x \succsim y$ or $y \succsim x$ (or both). No “I can’t decide.”
2. **Transitivity**: $x \succsim y$ and $y \succsim z \Rightarrow x \succsim z$. Prevents preference cycles.
3. **Continuity**: The sets $\{y : y \succsim x\}$ and $\{y : x \succsim y\}$ are closed. Ensures utility is a continuous function.
4. **Monotonicity (Non-satiation)**: More is better. $x \gg y \Rightarrow x \succ y$.

Utility Representation Theorem (Debreu): If \succsim is complete, transitive, and continuous on \mathbb{R}_+^n , then there exists a continuous function $u : \mathbb{R}_+^n \rightarrow \mathbb{R}$ such that $x \succsim y \iff u(x) \geq u(y)$.

TIER 3 — Advanced (Varian/Graduate): WARP and Revealed Preference

The **Weak Axiom of Revealed Preference (WARP)**: If x is chosen when y was affordable, then whenever y is chosen, x must not be affordable.

Formally: if $p \cdot x(p, m) \leq m$ (x is chosen at prices p) and $p' \cdot x(p, m) \leq m'$, then $p' \cdot x(p', m') \leq m'$ cannot hold with $x(p', m') \neq x(p, m)$ unless $x(p, m)$ was not affordable at (p', m') .

WARP is equivalent to the Slutsky matrix being negative semidefinite — the substitution effect always moves demand in the opposite direction of a price change.

1.1 Convexity and Its Economic Meaning

TIER 2 — Intermediate (Econ 101A): Convexity of Preferences

Preferences are **convex** if for any $x \sim y$, all convex combinations $\lambda x + (1 - \lambda)y \succsim x$ for $\lambda \in [0, 1]$. Geometrically: indifference curves are bowed toward the origin.

Economic intuition: Consumers prefer variety — a mix of two equally-good bundles is at least as good as either extreme.

Strict convexity \Rightarrow unique utility-maximizing bundle at any interior budget. This gives us

well-defined demand functions.

Quasi-concavity of u is the mathematical equivalent: $u(\lambda x + (1 - \lambda)y) \geq \min\{u(x), u(y)\}$ for all $\lambda \in [0, 1]$.

2. Utility Functions & Indifference Curves

2.1 Standard Functional Forms

TIER 2 — Intermediate (Econ 101A): The Four Canonical Forms

| Type | Form | MRS | Elast. Sub. (σ) | Shape |
|--------------|-------------------------|--------------------------------|--------------------------|------------------|
| Cobb-Douglas | $x^\alpha y^{1-\alpha}$ | $\frac{\alpha y}{(1-\alpha)x}$ | 1 | Smooth hyperbola |
| Perfect Subs | $ax + by$ | a/b (const) | ∞ | Straight lines |
| Perfect Comp | $\min(ax, by)$ | undefined | 0 | L-shaped |
| Quasilinear | $v(x) + y$ | $v'(x)$ | varies | Parallel curves |

TIER 3 — Advanced (Varian/Graduate): CES Utility and the Elasticity of Substitution

The CES (Constant Elasticity of Substitution) utility function nests all cases:

$$u(x, y) = \left[\alpha x^{(\sigma-1)/\sigma} + (1-\alpha)y^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)}$$

where σ is the elasticity of substitution.

- $\sigma \rightarrow 1$: Cobb-Douglas
- $\sigma \rightarrow \infty$: Perfect substitutes
- $\sigma \rightarrow 0$: Perfect complements (Leontief)

The MRS at any point equals $\alpha/(1-\alpha) \cdot (y/x)^{1/\sigma}$, and $\sigma = -\frac{d \ln(x/y)}{d \ln(MRS)}$.

EXAM WARNING: Ordinal vs. Cardinal Utility

Utility is **ordinal** — only the ranking matters, not the level. Any strictly increasing transformation $f(u)$ represents the same preferences. This means:

- $u = xy$ and $u = \ln x + \ln y$ are the same preferences
- You **cannot** compare utility across consumers
- Marginal utilities are **not** invariant to monotone transforms; MRS **is**

Part II

Consumer Theory

3. Marshallian Demand: The Utility Maximization Problem

TIER 1 — Principles (Econ 100A): The Budget Constraint

With income m and prices (p_x, p_y) , the budget set is $\{(x, y) : p_x x + p_y y \leq m\}$. The budget line has slope $-p_x/p_y$ and intercepts m/p_x (horizontal) and m/p_y (vertical). A price increase rotates the budget line inward; an income increase shifts it out.

TIER 2 — Intermediate (Econ 101A): The UMP: Setting Up the Lagrangian

$$\max_{x,y} u(x, y) \quad \text{s.t.} \quad p_x x + p_y y = m$$

$$\mathcal{L} = u(x, y) - \lambda(p_x x + p_y y - m)$$

First-Order Conditions:

$$\frac{\partial \mathcal{L}}{\partial x} = u_x - \lambda p_x = 0 \implies \frac{u_x}{p_x} = \lambda$$

$$\frac{\partial \mathcal{L}}{\partial y} = u_y - \lambda p_y = 0 \implies \frac{u_y}{p_y} = \lambda$$

Setting equal: $\frac{u_x}{u_y} = \frac{p_x}{p_y} \iff \text{MRS}_{xy} = \frac{p_x}{p_y}$

This is the tangency condition: the rate at which the consumer is willing to trade equals the rate at which the market requires trade.

TIER 2 — Intermediate (Econ 101A): Marshallian Demand for Cobb-Douglas

Let $u = x^\alpha y^{1-\alpha}$. FOCs give:

$$\frac{u_x}{u_y} = \frac{\alpha y}{(1-\alpha)x} = \frac{p_x}{p_y} \implies p_y y = \frac{1-\alpha}{\alpha} p_x x$$

Substituting into the budget constraint $p_x x + p_y y = m$:

$$p_x x + \frac{1-\alpha}{\alpha} p_x x = m \implies \frac{p_x x}{\alpha} = m$$

$$x^*(p_x, p_y, m) = \frac{\alpha m}{p_x}, \quad y^*(p_x, p_y, m) = \frac{(1-\alpha)m}{p_y}$$

Income-share rule: Cobb-Douglas consumers always spend fraction α on x and $(1-\alpha)$ on y , regardless of prices.

TIER 2 — Intermediate (Econ 101A): Corner Solutions

For perfect substitutes $u = ax + by$:

$$\text{If } \frac{a}{b} > \frac{p_x}{p_y} : \text{ buy only } x \implies x^* = m/p_x, y^* = 0$$

$$\text{If } \frac{a}{b} < \frac{p_x}{p_y} : \text{ buy only } y \implies x^* = 0, y^* = m/p_y$$

For perfect complements $u = \min(x/a, y/b)$: optimum at kink: $x/a = y/b$, so $x^* = \frac{am}{ap_x + bp_y}$,
 $y^* = \frac{bm}{ap_x + bp_y}$.

TIER 3 — Advanced (Varian/Graduate): The Indirect Utility Function and Roy's Identity

Substituting Marshallian demands back into utility gives the **indirect utility function**:

$$v(p_x, p_y, m) = u(x^*(p, m), y^*(p, m))$$

For Cobb-Douglas: $v(p, m) = \left(\frac{\alpha m}{p_x}\right)^\alpha \left(\frac{(1-\alpha)m}{p_y}\right)^{1-\alpha} = m \cdot \frac{\alpha^\alpha (1-\alpha)^{1-\alpha}}{p_x^\alpha p_y^{1-\alpha}}$

Roy's Identity recovers Marshallian demand from v :

$$x^*(p, m) = -\frac{\partial v / \partial p_x}{\partial v / \partial m}$$

This is crucial for welfare analysis and is a direct consequence of the Envelope Theorem applied to the UMP.

4. Duality: The Expenditure Function & Hicksian Demand

TIER 1 — Principles (Econ 100A): The Expenditure Minimization Problem (EMP)

The dual of utility maximization: find the minimum cost to achieve a target utility level \bar{u} :

$$\min_{x,y} p_x x + p_y y \quad \text{s.t.} \quad u(x, y) \geq \bar{u}$$

The solution is **Hicksian (compensated) demand** $h_x(p_x, p_y, \bar{u})$, which holds utility constant while prices change. The minimum cost is the **expenditure function** $e(p_x, p_y, \bar{u})$.

TIER 2 — Intermediate (Econ 101A): EMP Lagrangian and Hicksian Demand

$$\mathcal{L} = p_x x + p_y y - \mu(u(x, y) - \bar{u})$$

FOCs: $p_x = \mu u_x$, $p_y = \mu u_y \implies$ same tangency condition $MRS = p_x/p_y$. The difference from UMP is we fix u and minimize cost instead of fixing cost and maximizing u .

For Cobb-Douglas $u = x^\alpha y^{1-\alpha}$:

From tangency: $y = \frac{1-\alpha}{\alpha} \cdot \frac{p_x}{p_y} \cdot x$. Substituting into $x^\alpha y^{1-\alpha} = \bar{u}$:

$$h_x(p, \bar{u}) = \bar{u} \left(\frac{p_y}{\alpha} \cdot \frac{\alpha}{p_x}\right)^{1-\alpha} \cdot (\text{algebra}) = \bar{u} \cdot \left(\frac{\alpha p_y}{(1-\alpha)p_x}\right)^{1-\alpha}$$

$$e(p, \bar{u}) = \bar{u} \cdot \frac{p_x^\alpha p_y^{1-\alpha}}{\alpha^\alpha (1-\alpha)^{1-\alpha}}$$

TIER 3 — Advanced (Varian/Graduate): Shephard's Lemma

The expenditure function's partial derivative recovers Hicksian demand:

$$\frac{\partial e(p, \bar{u})}{\partial p_i} = h_i(p, \bar{u})$$

Proof sketch: By the Envelope Theorem applied to the EMP, the optimal-value derivative equals the constraint-evaluated derivative. Since p_i appears linearly in cost and not in the constraint, $\partial e / \partial p_i = x_i^* |_{\text{optimum}} = h_i$.

Why it matters: Given $e(p, \bar{u})$, you can recover all Hicksian demands without solving any optimization. Combined with $e(p, v(p, m)) = m$ (the duality identity), you can move freely between UMP and EMP.

KEY RESULT: The Duality Map

$$\text{UMP: } \max u \text{ s.t. } px = m \quad \longleftrightarrow \quad \text{EMP: } \min px \text{ s.t. } u \geq \bar{u}$$

$$\text{Solution: } x^*(p, m)$$

$$\text{Solution: } h(p, \bar{u})$$

$$\text{Value fn: } v(p, m)$$

$$\text{Value fn: } e(p, \bar{u})$$

$$v(p, e(p, \bar{u})) = \bar{u} \quad e(p, v(p, m)) = m$$

$$x^*(p, m) = h(p, v(p, m)) \quad h(p, \bar{u}) = x^*(p, e(p, \bar{u}))$$

5. The Slutsky Equation: Decomposing Price Effects

TIER 1 — Principles (Econ 100A): Income and Substitution Effects

When p_x rises, two things happen simultaneously:

- **Substitution effect (SE):** x is now relatively more expensive; substitute toward y . Always negative (for own-price changes).
- **Income effect (IE):** The price rise erodes purchasing power; real income falls. Sign depends on whether x is normal or inferior.

For a **Giffen good**: IE is so large and positive (strongly inferior) that it overwhelms the SE, making demand curve upward-sloping.

TIER 2 — Intermediate (Econ 101A): The Slutsky Equation (Mathematical Derivation)

Start from the identity: $x^*(p, e(p, \bar{u})) = h(p, \bar{u})$

Differentiate both sides with respect to p_x :

$$\frac{\partial x^*}{\partial p_x} + \frac{\partial x^*}{\partial m} \cdot \frac{\partial e}{\partial p_x} = \frac{\partial h}{\partial p_x}$$

Since $\partial e / \partial p_x = h_x = x^*$ (Shephard's Lemma at the same point):

$$\frac{\partial x^*}{\partial p_x} = \underbrace{\frac{\partial h}{\partial p_x}}_{\text{SE} \leq 0} - x^* \cdot \underbrace{\frac{\partial x^*}{\partial m}}_{\text{IE}}$$

The total price effect equals the (compensated) substitution effect minus the income effect.

TIER 2 — Intermediate (Econ 101A): Slutsky Decomposition: Numerical Example

$u = x^{1/2}y^{1/2}$, $m = 20$, $p_y = 1$, $p_x : 1 \rightarrow 4$.

Marshallian: $x^* = m/(2p_x)$

Initial: $x^* = 20/2 = 10$. **Final:** $x^* = 20/8 = 2.5$.

Total Effect (TE): $\Delta x = 2.5 - 10 = -7.5$

Compensated budget: $m' = e(4, 1, u_0)$. First find $u_0 = (10)^{1/2}(10)^{1/2} = 10$. Then $e = u_0 \cdot 2\sqrt{p_x p_y} = 10 \cdot 2\sqrt{4} = 40$. At new prices with $m' = 40$: $x^H = m'/(2p_x) = 40/8 = 5$.

SE: $5 - 10 = -5$. **IE:** $2.5 - 5 = -2.5$.

Check: $-5 + (-2.5) = -7.5 \checkmark$

Normal good ($\partial x^*/\partial m > 0$), so IE reinforces SE.

TIER 3 — Advanced (Varian/Graduate): The Slutsky Matrix

For n goods, define the Slutsky matrix S with entries $S_{ij} = \partial h_i / \partial p_j$.

Properties:

- **Symmetry:** $S_{ij} = S_{ji}$ (Young's theorem on $e(p, \bar{u})$)
- **Negative semidefinite:** $v^\top S v \leq 0$ for all v (follows from cost minimization)
- **Homogeneity:** $\sum_j S_{ij} p_j = 0$ for all i (Euler's theorem on h)

The NSD property is equivalent to WARP for two goods. The SE is $S_{ii} \leq 0$ — own-price Hicksian demand slopes down, always.

TIER 2 — Intermediate (Econ 101A): Welfare Measurement: EV, CV, and CS

When price changes from p^0 to p^1 :

- **Compensating Variation (CV):** Income change that leaves consumer at old utility u^0 after price change. $CV = e(p^1, u^0) - e(p^0, u^0) = e(p^1, u^0) - m$
- **Equivalent Variation (EV):** Income change that achieves same welfare change at old prices. $EV = e(p^1, u^1) - e(p^0, u^1) = m - e(p^0, u^1)$
- **Consumer Surplus (ΔCS):** Area under Marshallian demand curve. Approximation when income effects are small.

Order for a price decrease: $EV \geq \Delta CS \geq CV$ for normal goods.

6. Elasticity, Consumer Surplus & Tax Incidence

TIER 2 — Intermediate (Econ 101A): Elasticity Definitions

| Elasticity | Formula | Interpretation |
|-----------------------------|---|--|
| Own-price (ε) | $\frac{\partial x^*}{\partial p_x} \cdot \frac{p_x}{x^*}$ | % change in x per 1% rise in p_x |
| Cross-price | $\frac{\partial x^*}{\partial p_y} \cdot \frac{p_y}{x^*}$ | Positive: substitutes; Negative: complements |
| Income (η) | $\frac{\partial x^*}{\partial m} \cdot \frac{m}{x^*}$ | > 1 : luxury; $0 < \eta < 1$: necessity; < 0 : inferior |

Cobb-Douglas: All own-price elasticities = -1 , all income elasticities = 1 .

TIER 2 — Intermediate (Econ 101A): Tax Incidence via IFT

Market clearing: $D(p) = S(p)$. A specific tax t on producers shifts supply: $S(p - t)$.
 Market clearing with tax: $D(p^*) = S(p^* - t)$. Define $F(p^*, t) = D(p^*) - S(p^* - t) = 0$.
 By IFT:

$$\frac{dp^*}{dt} = -\frac{F_t}{F_{p^*}} = -\frac{S'(p^* - t)}{D'(p^*) - S'(p^* - t)} = \frac{S'}{S' - D'} = \frac{\varepsilon_S}{\varepsilon_S - \varepsilon_D}$$

where $\varepsilon_S > 0$ and $\varepsilon_D < 0$. Since $\varepsilon_S - \varepsilon_D > 0$: **buyer price rises, but by less than t** .
 Consumer bears fraction $\varepsilon_S/(\varepsilon_S - \varepsilon_D)$ of tax; producer bears $-\varepsilon_D/(\varepsilon_S - \varepsilon_D)$.

7. Intertemporal Choice & The Euler Equation**TIER 1 — Principles (Econ 100A): Two-Period Framework**

Consumer lives two periods. Earns (y_0, y_1) , chooses (c_0, c_1) . Interest rate r ; borrows/saves at rate r .

Budget constraint: $c_0 + \frac{c_1}{1+r} = y_0 + \frac{y_1}{1+r} = W$ (present value of wealth)

Preferences: $U(c_0, c_1) = u(c_0) + \beta u(c_1)$ where $\beta = 1/(1 + \rho)$ is the discount factor and ρ is the pure rate of time preference.

TIER 2 — Intermediate (Econ 101A): The Euler Equation

Lagrangian: $\mathcal{L} = u(c_0) + \beta u(c_1) - \lambda \left(c_0 + \frac{c_1}{1+r} - W \right)$

FOCs:

$$u'(c_0) = \lambda, \quad \beta(1+r)u'(c_1) = \lambda$$

$$\boxed{u'(c_0) = \beta(1+r)u'(c_1)}$$

Interpretation: At the optimum, the marginal utility of consuming today equals the discounted marginal utility of consuming tomorrow. Consume more today if $\beta(1+r) < 1$ (impatient or low return), and save if $\beta(1+r) > 1$.

Log utility ($u = \ln c$): $u'(c) = 1/c$, so $1/c_0 = \beta(1+r)/c_1 \implies c_1/c_0 = \beta(1+r)$. Consumption growth equals $\beta(1+r)$.

TIER 3 — Advanced (Varian/Graduate): Consumption Smoothing and Permanent Income

With CRRA utility $u(c) = c^{1-\sigma}/(1-\sigma)$, $u'(c) = c^{-\sigma}$:

$$c_0^{-\sigma} = \beta(1+r)c_1^{-\sigma} \implies \frac{c_1}{c_0} = [\beta(1+r)]^{1/\sigma}$$

The elasticity of intertemporal substitution (EIS) is $1/\sigma$. Higher EIS (σ small) \Rightarrow more responsive consumption growth to changes in r .

Permanent Income Hypothesis: Consumers maximize utility subject to the constraint that the present value of consumption equals the present value of income. Only *permanent* income shocks affect consumption; *transitory* shocks are smoothed.

8. Labor Supply

TIER 2 — Intermediate (Econ 101A): The Labor-Leisure Tradeoff

Consumer endowed with T units of time. Chooses leisure ℓ and consumption c .

- Labor: $L = T - \ell$
- Budget constraint: $c = w(T - \ell) + V$ where w is the wage and V is non-labor income
- UMP: $\max U(c, \ell)$ s.t. $c + w\ell = wT + V \equiv M$ (full income)

Think of leisure as a “good” with price w (the opportunity cost of not working).

TIER 2 — Intermediate (Econ 101A): Labor Supply and the Backward-Bending Curve

FOC: $MRS_{c,\ell} = U_\ell/U_c = w$. Optimal leisure from Marshallian demand; labor supply $L^* = T - \ell^*$.

Effect of wage increase ($w \uparrow$):

- **SE:** Leisure is more expensive \rightarrow substitute toward work. $\partial\ell/\partial w|_{\bar{u}} < 0$, so L^S rises.
- **IE:** Higher wage \rightarrow higher real income \rightarrow demand more leisure (if normal). L^S falls.

At low wages, SE dominates: supply curve slopes up. At high wages, IE dominates: **backward-bending** labor supply curve.

9. Risk & Expected Utility Theory

TIER 1 — Principles (Econ 100A): Expected Value vs. Expected Utility

A lottery pays x_1 with probability p and x_2 with probability $1 - p$.

- **Expected Value (EV):** $E[X] = px_1 + (1-p)x_2$ — the average payoff
- **Expected Utility (EU):** $E[U(X)] = pu(x_1) + (1-p)u(x_2)$ — utility of average vs. average utility

Risk aversion (concave u): $E[U(X)] < U(E[X])$ by Jensen’s Inequality. The consumer prefers the sure thing $E[X]$ over the gamble.

TIER 2 — Intermediate (Econ 101A): Arrow-Pratt Risk Aversion Measures

- **Absolute Risk Aversion (ARA):** $A(w) = -\frac{u''(w)}{u'(w)}$
 - CARA (Constant ARA): $u(w) = -e^{-\gamma w}$, $A(w) = \gamma$
- **Relative Risk Aversion (RRA):** $R(w) = -\frac{w \cdot u''(w)}{u'(w)}$
 - CRRA: $u(w) = w^{1-\rho}/(1-\rho)$, $R(w) = \rho$

ARA measures willingness to pay to avoid a fixed-size gamble; RRA measures it for proportional gambles.

TIER 2 — Intermediate (Econ 101A): Certainty Equivalent for CARA-Normal

Let $u(w) = -e^{-\gamma w}$ and $\tilde{W} = W_0 + \tilde{X}$ where $\tilde{X} \sim N(\mu, \sigma^2)$.

Step 1: Compute EU using MGF.

$$E[u(\tilde{W})] = -E[e^{-\gamma \tilde{W}}] = -e^{-\gamma W_0} \cdot E[e^{-\gamma \tilde{X}}]$$

Since $E[e^{t\tilde{X}}] = e^{t\mu + t^2\sigma^2/2}$ (MGF of Normal), set $t = -\gamma$:

$$E[u(\tilde{W})] = -e^{-\gamma W_0} \cdot e^{-\gamma\mu + \gamma^2\sigma^2/2} = -e^{-\gamma(W_0 + \mu - \gamma\sigma^2/2)}$$

Step 2: Set equal to $u(\text{CE})$.

$$-e^{-\gamma \text{CE}} = -e^{-\gamma(W_0 + \mu - \gamma\sigma^2/2)}$$

$$\boxed{\text{CE} = W_0 + \mu - \frac{\gamma}{2}\sigma^2}$$

The certainty equivalent equals expected wealth minus a **risk premium** of $\frac{\gamma}{2}\sigma^2$.

TIER 3 — Advanced (Varian/Graduate): Jensen's Inequality and Risk Premia

For any random variable \tilde{X} and concave function u :

$$E[u(\tilde{X})] \leq u(E[\tilde{X}])$$

The **risk premium** π solves $u(E[\tilde{X}] - \pi) = E[u(\tilde{X})]$, i.e., $\text{CE} = E[\tilde{X}] - \pi$.

For small gambles, the Pratt approximation gives: $\pi \approx \frac{1}{2}A(w) \cdot \text{Var}(\tilde{X})$.

For CARA-Normal: exact result is $\pi = \frac{\gamma}{2}\sigma^2$, confirming the Pratt approximation is exact in this case.

Part III**Producer Theory****10. Production, Cost Minimization & Conditional Factor Demands**

TIER 1 — Principles (Econ 100A): Production Functions and Returns to Scale

$f(K, L)$ maps inputs to output. Key concept: **returns to scale**.

- CRS: $f(\lambda K, \lambda L) = \lambda f(K, L)$ — e.g., $f = K^\alpha L^\beta$ with $\alpha + \beta = 1$
- IRS: $f(\lambda K, \lambda L) > \lambda f(K, L)$ — $\alpha + \beta > 1$
- DRS: $f(\lambda K, \lambda L) < \lambda f(K, L)$ — $\alpha + \beta < 1$

MRTS: $\text{MRTS}_{LK} = MP_L/MP_K$ — the rate at which labor substitutes for capital along an isoquant.

TIER 2 — Intermediate (Econ 101A): Cost Minimization Problem (CMP)

$$\min_{K, L} wL + rK \quad \text{s.t.} \quad f(K, L) = q$$

$$\mathcal{L} = wL + rK - \lambda(f(K, L) - q)$$

FOCs: $w = \lambda f_L, r = \lambda f_K \implies \text{MRTS}_{LK} = \frac{f_L}{f_K} = \frac{w}{r}$

Same geometric insight: isoquant tangent to isocost. Solution gives **conditional factor demands** $L^c(w, r, q)$, $K^c(w, r, q)$ and **cost function** $c(w, r, q)$.

Shephard's Lemma for costs:

$$\frac{\partial c(w, r, q)}{\partial w} = L^c(w, r, q), \quad \frac{\partial c(w, r, q)}{\partial r} = K^c(w, r, q)$$

TIER 2 — Intermediate (Econ 101A): Cost Function for Cobb-Douglas

$f = K^\alpha L^\beta$, target output q . From tangency: $K = \frac{\alpha w}{\beta r} \cdot L$.

Substituting into constraint and solving:

$$c(w, r, q) = \kappa \cdot w^{\beta/(\alpha+\beta)} \cdot r^{\alpha/(\alpha+\beta)} \cdot q^{1/(\alpha+\beta)}$$

where $\kappa = (\alpha/\beta)^{-\alpha/(\alpha+\beta)} + (\alpha/\beta)^{\beta/(\alpha+\beta)}$ is a constant.

For CRS ($\alpha + \beta = 1$): $c = \kappa \cdot w^\beta r^\alpha \cdot q$ — cost is linear in output (constant MC).

TIER 3 — Advanced (Varian/Graduate): Properties of Cost Functions

A well-behaved cost function satisfies:

1. **Non-decreasing** in (w, r, q)
2. **Homogeneous of degree 1** in (w, r) : $c(\lambda w, \lambda r, q) = \lambda c(w, r, q)$
3. **Concave** in (w, r) : $\partial^2 c / \partial w^2 \leq 0$ (input demand curves slope down)
4. **Continuous** in all arguments

The Hessian of c w.r.t. (w, r) is NSD — this is the matrix form of the concavity condition and captures how factor demands respond to price changes.

11. Profit Maximization & Competitive Supply

TIER 2 — Intermediate (Econ 101A): The Profit Maximization Problem

Short-run (K fixed at \bar{K}):

$$\max_L \pi = pf(L, \bar{K}) - wL - r\bar{K}$$

FOC: $p \cdot MP_L = w \implies MP_L = w/p$ (real wage). Labor demand falls as p falls or w rises.

SOC: $p \cdot f_{LL} < 0 \implies f_{LL} < 0$ — diminishing marginal product. This ensures the FOC is a maximum.

Long-run:

$$\max_{K,L} \pi = pf(K, L) - wL - rK$$

FOCs: $p \cdot MP_L = w$ and $p \cdot MP_K = r$. These simultaneously pin down L^* and K^* .

TIER 2 — Intermediate (Econ 101A): Supply Function and Comparative Statics via IFT

The FOC $F(L^*, p, w) \equiv p \cdot f_L(L^*) - w = 0$ implicitly defines $L^*(p, w)$.

By IFT:

$$\frac{\partial L^*}{\partial p} = -\frac{F_p}{F_L} = -\frac{f_L(L^*)}{p \cdot f_{LL}(L^*)}$$

Since $f_L > 0$ and $f_{LL} < 0$: $\partial L^*/\partial p > 0$. Higher output price \rightarrow higher labor demand.

Output supply: $y^*(p) = f(L^*(p))$. By the Envelope Theorem: $\partial \pi^*/\partial p = y^*$ (Hotelling's Lemma).

Supply slopes up in p .

12. Long-Run Competitive Equilibrium

TIER 1 — Principles (Econ 100A): Zero-Profit Condition

In the long run, free entry/exit drives economic profits to zero. If $\pi > 0$, firms enter; if $\pi < 0$, they exit. The long-run equilibrium price equals the minimum of the long-run average cost (LRAC):

$$p^* = \min_q \frac{c(w, r, q)}{q} = AC^*$$

The long-run supply curve is perfectly elastic at p^* .

TIER 2 — Intermediate (Econ 101A): Long-Run Equilibrium Analysis

Given a downward-sloping market demand $Q^D(p)$ and a U -shaped LRAC:

1. Each firm produces q^* where $p = MC = AC$ (tangency at minimum LRAC). 2. Number of firms: $n^* = Q^D(p^*)/q^*$. 3. A demand shock shifts Q^D : in short run, p rises, $\pi > 0$; in long run, entry restores $p = p^*$ at higher n^* .

The long run supply curve traces out: $p = AC^{min}$, which shifts only if input prices change.

Part IV

Market Equilibrium & Comparative Statics

13. Competitive Equilibrium & the IFT

TIER 2 — Intermediate (Econ 101A): Equilibrium and the IFT: Four-Step Method

Setting: Market-clearing condition $D(p^*; \theta) = S(p^*; \theta)$ where θ is an exogenous parameter.

Define $F(p^*, \theta) \equiv D(p^*; \theta) - S(p^*; \theta) = 0$.

Step 1: Verify IFT applies: $F_p = D_p - S_p \neq 0$. Since $D_p < 0$ and $S_p > 0$, we have $F_p < 0$. ✓

Step 2: Apply IFT:

$$\frac{dp^*}{d\theta} = -\frac{F_\theta}{F_p} = -\frac{D_\theta - S_\theta}{D_p - S_p}$$

Step 3: Compute F_θ by differentiating market clearing with respect to θ .

Step 4: Sign the result using economic restrictions on D and S .

TIER 2 — Intermediate (Econ 101A): Example: Specific Tax on Producers

Tax t enters as $S(p - t)$ (producer receives $p - t$). Clearing: $D(p) = S(p - t)$.

$F(p, t) = D(p) - S(p - t) = 0$

$F_p = D'(p) - S'(p - t)$, $F_t = S'(p - t)$

$$\frac{dp^*}{dt} = -\frac{S'}{D' - S'} = \frac{S'}{S' - D'}$$

Since $S' > 0$ and $S' - D' > 0$: consumer price rises. The fraction $S'/(S' - D')$ is between 0 and 1 — consumers bear fraction proportional to supply elasticity.

Part V

Market Power

14. Monopoly & Price Discrimination

TIER 1 — Principles (Econ 100A): Monopoly Basics

A monopolist faces the entire market demand $P(Q)$ (inverse demand). Revenue: $R(Q) = P(Q) \cdot Q$. The monopolist maximizes profit:

$$\max_Q P(Q) \cdot Q - C(Q)$$

FOC: $\underbrace{P(Q) + P'(Q) \cdot Q}_{MR} = MC(Q)$

TIER 2 — Intermediate (Econ 101A): MR and the Lerner Index

$MR = P \left(1 + \frac{1}{\varepsilon}\right)$ where $\varepsilon < 0$ is the price elasticity.

Setting $MR = MC$: $P \left(1 + \frac{1}{\varepsilon}\right) = MC \implies \frac{P - MC}{P} = -\frac{1}{\varepsilon}$

The **Lerner Index** $L = (P - MC)/P = 1/|\varepsilon|$ measures market power. Competitive: $L = 0$; monopoly: $L > 0$.

DWL: $\int_{Q^M}^{Q^C} [P(Q) - MC] dQ$ — area of the welfare triangle between Q^M and Q^C .

TIER 2 — Intermediate (Econ 101A): Third-Degree Price Discrimination

Monopolist separates market into groups with different elasticities. Profit:

$$\max_{Q_1, Q_2} P_1(Q_1)Q_1 + P_2(Q_2)Q_2 - C(Q_1 + Q_2)$$

FOCs: $MR_1(Q_1) = MC$ and $MR_2(Q_2) = MC$.

Equating: $MR_1 = MR_2 = MC$ (allocate output so marginal revenues are equal).

Inverse elasticity rule: $P_i \left(1 + \frac{1}{\varepsilon_i}\right) = MC \implies \frac{P_1}{P_2} = \frac{1 + 1/\varepsilon_2}{1 + 1/\varepsilon_1}$

Higher price charged to inelastic market ($|\varepsilon|$ small). Condition for market to be served: $MR_i \geq MC$ at $Q_i = 0$ (i.e., $P_i(0) \geq MC$).

15. Oligopoly & Game Theory**TIER 2 — Intermediate (Econ 101A): Cournot Nash Equilibrium**

n firms, homogeneous good, simultaneous quantity competition. Firm i maximizes:

$$\pi_i = P(Q) \cdot q_i - c_i(q_i), \quad Q = \sum_j q_j$$

FOC (Best-response function): $P(Q) + P'(Q) \cdot q_i = c'_i(q_i)$

Linear demand $P = a - bQ$, **symmetric** $MC = c$:

$$a - b(q_i + \sum_{j \neq i} q_j) - bq_i - c = 0 \implies q_i = \frac{a - c}{2b} - \frac{\sum_{j \neq i} q_j}{2}$$

$$\text{Symmetric NE: } q^* = \frac{a - c}{b(n + 1)}, \quad P^* = \frac{a + nc}{n + 1}, \quad \pi^* = b(q^*)^2.$$

As $n \rightarrow \infty$: $P^* \rightarrow c = MC$. Cournot \rightarrow competitive equilibrium.

TIER 2 — Intermediate (Econ 101A): Stackelberg: First-Mover Advantage

Firm 1 moves first (leader), Firm 2 observes and responds. Solve by backward induction.

Step 1 (Firm 2's reaction): $q_2^R(q_1) = \frac{a - c}{2b} - \frac{q_1}{2}$

Step 2 (Firm 1 internalizes this):

$$\max_{q_1} (a - b(q_1 + q_2^R(q_1)))q_1 - cq_1 = \left(\frac{a - c}{2} - \frac{bq_1}{2}\right)q_1 - \text{const}$$

$$q_1^* = \frac{a-c}{2b} \text{ (Cournot } \times 2), q_2^* = \frac{a-c}{4b}, P^* = \frac{a+3c}{4}.$$

First-mover produces more, earns more ($\pi_1 > \pi_2$); consumers benefit vs. Cournot ($P^{Stack} < P^{Cournot}$).

TIER 2 — Intermediate (Econ 101A): Bertrand Competition

Firms compete on **price** (not quantity). Homogeneous product, identical $MC = c$.

Bertrand Paradox: Unique NE is $p_1 = p_2 = c = MC$, so $\pi_1 = \pi_2 = 0$. Even two firms are enough for competitive outcome.

Logic: If $p_i > c$, firm j can undercut by ϵ and capture entire market. Only $p = c$ is stable.

Bertrand vs. Cournot: Same setup, different instruments. Cournot: $P > MC$. Bertrand: $P = MC$. Bertrand is the “more competitive” outcome.

TIER 3 — Advanced (Varian/Graduate): Nash Equilibrium: General Definition

A strategy profile (s_1^*, \dots, s_n^*) is a Nash Equilibrium if:

$$\pi_i(s_i^*, s_{-i}^*) \geq \pi_i(s_i, s_{-i}^*) \quad \forall s_i, \forall i$$

No player can profitably deviate holding others’ strategies fixed. NE is a fixed point of the best-response correspondence. By Nash’s theorem (1950), every finite game has at least one NE in mixed strategies.

Part VI

General Equilibrium & Welfare

16. Pure Exchange Economy & Walrasian Equilibrium

TIER 1 — Principles (Econ 100A): The Edgeworth Box

Two consumers (A, B), two goods (x, y), fixed endowments (ω^A, ω^B) with $\omega^A + \omega^B = \bar{\omega}$. Every interior point represents a feasible allocation. Pareto improvements exist when indifference curves are not tangent.

TIER 2 — Intermediate (Econ 101A): Walrasian Equilibrium: The 4-Step Method

Step 1: Derive Marshallian demands for each consumer.

Wealth of consumer i : $m^i = p_x \omega_x^i + p_y \omega_y^i$. Solve UMP given (p_x, p_y, m^i) .

Step 2: Normalize prices. Set $p_y = 1$ (numeraire). Only relative prices p_x/p_y matter by Walras’ Law.

Step 3: Market clearing for one good.

$$x^{A*}(p_x, 1, m^A) + x^{B*}(p_x, 1, m^B) = \bar{\omega}_x$$

Solve for p_x^* .

Step 4: Recover allocations. Plug p_x^* into demands; verify y market also clears (Walras' Law guarantees this if x market clears).

TIER 2 — Intermediate (Econ 101A): Example: Cobb-Douglas + Leontief

$U_A = x_A^{1/2} y_A^{1/2}$, $\omega_A = (4, 0)$; $U_B = \min(x_B, y_B)$, $\omega_B = (0, 2)$.

Normalize $p_y = 1$.

$m^A = 4p_x$, $m^B = 2$. Marshallian demands:

$$x^A = \frac{m^A}{2p_x} = 2, \quad y^A = \frac{m^A}{2} = 2p_x$$

$$x^B = y^B = \frac{m^B}{p_x + 1} = \frac{2}{p_x + 1}$$

Market clearing for x : $2 + \frac{2}{p_x + 1} = 4 \implies \frac{2}{p_x + 1} = 2 \implies p_x^* = 0???$

Wait: endowment is $\omega_A = (4, 0)$, $\omega_B = (0, 2)$, so $\bar{\omega}_x = 4$.

$2 + \frac{2}{p_x + 1} = 4 \implies \frac{2}{p_x + 1} = 2 \implies p_x + 1 = 1 \implies p_x = 0$.

If $p_x = 0$: $m^A = 0$, so $y^A = 0$; $m^B = 2$, $x^B = y^B = 2/1 = 2$. Check x : $0 + 2 = 2 \neq 4$.

Diagnosis: Consumer A has only x endowment; if $p_x = 0$, wealth=0, A can't buy anything. Need to re-examine.

Correct: $x^A = \frac{m^A}{2p_x} = \frac{4p_x}{2p_x} = 2$ (regardless of p_x !). $y^A = \frac{m^A}{2} = 2p_x$.

$x^B = y^B = \frac{2}{p_x + 1}$.

Clearing for x : $2 + \frac{2}{p_x + 1} = 4 \implies p_x^* = 0$.

At $p_x^* = 0$: $x^A = 2$, $y^A = 0$; $x^B = 2$, $y^B = 2$. But $y^A + y^B = 2 = \bar{\omega}_y \checkmark$, $x^A + x^B = 4 = \bar{\omega}_x \checkmark$.

Equilibrium: $p_x^* = 0$, $p_y = 1$. A gets $(2, 0)$; B gets $(2, 2)$.

Note: $p_x = 0$ is valid — free good x when A cares equally and B demands equal quantities.

TIER 2 — Intermediate (Econ 101A): Contract Curve & Pareto Optimality

A Pareto optimal allocation satisfies $MRS^A = MRS^B$ (for interior solutions). The **contract curve** is the locus of all PO allocations in the Edgeworth box.

For Cobb-Douglas: $MRS^A = \frac{\alpha_A y_A}{(1 - \alpha_A) x_A} = \frac{\alpha_B y_B}{(1 - \alpha_B) x_B} = MRS^B$

Substituting $x_B = \bar{\omega}_x - x_A$ and $y_B = \bar{\omega}_y - y_A$ gives the contract curve equation.

TIER 3 — Advanced (Varian/Graduate): The Two Welfare Theorems

First Welfare Theorem: Every Walrasian equilibrium allocation is Pareto optimal. (Markets are efficient — no planner can improve on the competitive outcome without making someone worse off.)

Proof: At WE, $MRS^A = p_x/p_y = MRS^B$ for all consumers. Hence the WE allocation lies on the contract curve.

Second Welfare Theorem: Any Pareto optimal allocation can be achieved as a WE after appropriate lump-sum redistribution of endowments. (Efficiency and equity can be separated.)

Caveat: 2WT requires convex preferences, complete markets, and the ability to implement lump-sum transfers — strong assumptions.

17. Robinson Crusoe Economy

TIER 2 — Intermediate (Econ 101A): Unifying Consumer & Producer Theory

One consumer, one firm (owned by consumer). Consumer maximizes $U(\ell, c)$; firm uses labor to produce consumption good $c = f(L)$.

The Planner's Problem: $\max_{\ell} U(\ell, f(T - \ell))$

FOC: $U_{\ell} + U_c f'(T - \ell)(-1) = 0 \implies \frac{U_{\ell}}{U_c} = f'(L)$

MRS between leisure and consumption equals marginal product of labor — the social optimum.

TIER 2 — Intermediate (Econ 101A): Decentralization: Why Markets Work

Firm: $\max_L pf(L) - wL$. FOC: $pf'(L^*) = w \implies f'(L^*) = w/p$.

Consumer: $\max_{\ell} U(\ell, c)$ s.t. $p\ell + wL = wT + \pi^*$. FOC: $U_{\ell}/U_c = w/p$.

Market Clearing: $L^* = T - \ell^*$ and $c^* = f(L^*)$.

Both set $MRS = w/p = f'(L^*)$. The decentralized equilibrium replicates the planner's solution — a proof of the **First Welfare Theorem** in a production economy.

Part VII

Market Failures

18. Externalities & Pigouvian Taxation

TIER 1 — Principles (Econ 100A): What Is an externality?

An externality occurs when one agent's action affects another's utility or profit without being mediated through prices. **Negative externality:** pollution, noise. **Positive externality:** education, vaccination.

Market fails because the producer/consumer does not internalize the full social cost (or benefit) of their action.

TIER 2 — Intermediate (Econ 101A): Deriving the Pigouvian Tax

Firm produces output q at private cost $C(q)$. Third party suffers damage $D(q)$.

Private FOC: $p = C'(q^P)$ — firm equates price to private MC.

Social FOC: $p = C'(q^S) + D'(q^S)$ — planner equates price to social MC = PMC + MED.

Result: $q^P > q^S$ (overproduction).

Pigouvian tax: Set $t^* = D'(q^S) = \text{MED}$ at social optimum. With tax, firm's FOC becomes:

$$p - t^* = C'(q) \implies p = C'(q) + t^* = C'(q^S) + D'(q^S) \implies q = q^S$$

Tax forces firm to internalize the externality.

DWL without tax: $\int_{q^S}^{q^P} [D'(q) - \underbrace{(p - C'(q))}_{\geq 0}] dq \geq 0$. The integral of MED over the excess output.

19. Public Goods & the Samuelson Condition

TIER 2 — Intermediate (Econ 101A): Non-Rival and Non-Excludable

A **public good** is (i) non-rival: one person's consumption does not reduce another's; (ii) non-excludable: cannot prevent non-payers from consuming.

Free-rider problem: Each consumer free-rides on others' provision. Private provision yields $Q < Q^{social}$.

Samuelson Condition: Pareto optimal provision equates sum of marginal benefits to marginal cost:

$$\sum_i MRS_i = \frac{MC}{p_x}$$

or, in a 2-person economy with MC_G : $MB_A + MB_B = MC_G$.

Compare to private good: each person separately faces $MB_i = p_G$, no summing.

Part VIII

Information, Incentives & Behavioral Economics

20. Asymmetric Information: Adverse Selection

TIER 1 — Principles (Econ 100A): The Lemons Problem (Akerlof 1970)

Sellers know car quality; buyers do not. If buyers offer average-quality price, only bad cars (“lemons”) are offered, lowering average quality, lowering price further — **market unravels**.

TIER 2 — Intermediate (Econ 101A): Adverse Selection: Formal Model

Two types of cars: low quality v_L (seller cost c_L), high quality v_H (seller cost c_H), with $v_H > v_L$ and $v_i > c_i$ (gains from trade).

Fraction q are high quality; buyers believe this but cannot distinguish.

Buyer's WTP: $\mu = qv_H + (1 - q)v_L$ (expected value at belief q).

Seller participation:

- High-quality sellers sell only if $\mu \geq c_H \iff q \geq \frac{c_H - v_L}{v_H - v_L}$
- Low-quality sellers always sell if $\mu \geq c_L$

Lemons equilibrium: If $q < \frac{c_H - v_L}{v_H - v_L}$, high-quality sellers exit, market price drops to v_L , only lemons trade. Even though $v_H > c_H$ (gains from trade in high-quality cars), they are not traded.

TIER 3 — Advanced (Varian/Graduate): Signaling and Screening

Signaling (Spence): High types can credibly signal quality via a costly, but differentially costly, signal (education). Separating equilibrium: education e^* satisfies $c_H(e^*) \leq \Delta w \leq c_L(e^*)$ where $\Delta w = w_H - w_L$.

Screening (Rothschild-Stiglitz): Uninformed principal offers a menu of contracts; informed agent self-selects. High-risk types get full insurance at high premium; low-risk types get partial insurance at low premium. IR and IC constraints bind.

21. Behavioral Economics**TIER 2 — Intermediate (Econ 101A): Time Inconsistency: Quasi-Hyperbolic Discounting**

Standard exponential discounting: $U = u_0 + \delta u_1 + \delta^2 u_2 + \dots$

Quasi-hyperbolic (β - δ) discounting:

$$U_t = u_t + \beta \sum_{s=t+1}^{\infty} \delta^{s-t} u_s, \quad \beta < 1$$

$\beta < 1$ creates present bias — extra impatience toward near-term tradeoffs.

Time inconsistency: Plans made at $t = 0$ for $t = 2$ vs. $t = 1$ are revised at $t = 1$. “I’ll start the diet tomorrow” — but tomorrow becomes today.

Naive vs. Sophisticated: Naive agents don’t anticipate future inconsistency. Sophisticated agents do (Strotz 1956) and may use commitment devices.

TIER 2 — Intermediate (Econ 101A): Prospect Theory (Kahneman-Tversky)

Key departures from expected utility:

1. **Reference dependence:** Utility defined over gains/losses from a reference point, not final wealth
2. **Loss aversion:** |utility of loss| > |utility of equal gain|. Typically $\lambda \approx 2.25$ in experiments.
3. **Diminishing sensitivity:** Value function concave over gains, convex over losses
4. **Probability weighting:** People overweight small probabilities, underweight large ones

Value function: $v(x) = x^\alpha$ for gains, $-\lambda(-x)^\alpha$ for losses, with $\lambda > 1$ and $0 < \alpha < 1$.

TIER 2 — Intermediate (Econ 101A): Nudges and Choice Architecture

Libertarian paternalism (Thaler-Sunstein): Preserve choice but design defaults to improve outcomes.

Key behavioral phenomena:

- **Default effect:** Opt-out vs. opt-in dramatically changes take-up (organ donation, 401k enrollment)
- **Anchoring:** Starting point influences judgment (salary negotiations, price estimates)
- **Framing:** Equivalent information presented differently leads to different choices (“95% fat

free” vs. “5% fat”)

- **Sunk cost fallacy:** Letting irrecoverable past costs influence forward-looking decisions

Policy implication: Pigouvian taxes address externalities; behavioral nudges address internalities (self-inflicted harms from bounded rationality).

Part IX

Appendix A: Master Formula Reference

A. Master Formula Cheat Sheet

MASTER FORMULA: Consumer Theory Identities

| Formula | Meaning |
|---|-----------------------------|
| $MRS = p_x/p_y$ | Optimality condition (UMP) |
| $MRTS = w/r$ | Optimality condition (CMP) |
| $\partial e/\partial p_i = h_i$ | Shephard's Lemma (consumer) |
| $\partial c/\partial w_i = x_i^c$ | Shephard's Lemma (firm) |
| $x^*(p, m) = -(\partial v/\partial p_x)/(\partial v/\partial m)$ | Roy's Identity |
| $\partial x^*/\partial p_x = \partial h/\partial p_x - x^* \cdot \partial x^*/\partial m$ | Slutsky Equation |
| $v(p, e(p, \bar{u})) = \bar{u}; e(p, v(p, m)) = m$ | Duality Identities |
| $u'(c_0) = \beta(1+r)u'(c_1)$ | Euler Equation |
| $CE = W_0 + \mu - \frac{\gamma}{2}\sigma^2$ | CARA-Normal CE |

MASTER FORMULA: Producer & Market Formulas

| Formula | Meaning |
|---|------------------------------------|
| $p = MC$ | Competitive FOC |
| $MR = MC$ | Monopoly/Oligopoly FOC |
| $MR = P(1 + 1/\varepsilon)$ | MR-Demand relationship |
| $(P - MC)/P = -1/\varepsilon = 1/ \varepsilon $ | Lerner Index |
| $dp^*/dt = S'/(S' - D')$ | Tax incidence (IFT) |
| $q^* = (a - c)/[b(n + 1)]$ | Cournot NE (n symmetric firms) |
| $q_1^{Stack} = (a - c)/(2b)$ | Stackelberg leader |
| $t^* = D'(q^S)$ | Optimal Pigouvian tax |
| $\sum_i MB_i = MC_G$ | Samuelson condition (public goods) |

MASTER FORMULA: General Equilibrium Formulas

| Formula | Meaning |
|--|------------------------------------|
| $MRS^A = MRS^B$ | Contract curve / Pareto optimality |
| $\sum_i x_i^* = \sum_i \omega_i$ | Market clearing |
| $\sum_i p_j(x_{ij}^* - \omega_{ij}) = 0$ | Walras' Law |
| $MRS = w/p = f'(L)$ | Robinson Crusoe equilibrium |
| $p \cdot f_L = w$ and $p \cdot f_K = r$ | Firm's profit-max FOCs |

B. Pattern Recognition Atlas

TIER 2 — Intermediate (Econ 101A): Trigger → Tool: The Exam Decoder

| When you see... | Deploy... |
|--------------------------------------|---|
| “Derive demand” | Lagrangian UMP → FOC → tangency + budget |
| “Hicksian demand” | EMP Lagrangian OR Shephard’s Lemma on $e(p, \bar{u})$ |
| “Decompose price effect” | Slutsky: $TE = SE + IE$; find h and x^* |
| “Welfare of price change” | CV/EV = change in expenditure function |
| “Optimal tax” | Pigouvian: set $t^* = MED$ at social optimum |
| “General equilibrium” | 4-step: demands → normalize → market clear → allocations |
| “Pareto optimal” | MRS equality + Contract curve |
| “Nash Equilibrium” | Best-response functions → solve simultaneously |
| “Prove monotone comparative static” | IFT on FOC: sign $-F_\theta/F_x$ |
| “CARA utility + Normal distribution” | MGF trick: $CE = \mu_W - \frac{\gamma}{2}\sigma_W^2$ |
| “Euler equation” | Intertemporal FOC: $u'(c_0) = \beta(1 + r)u'(c_1)$ |
| “Robinson Crusoe” | Planner = Firm + Consumer; decentralize; $MRS = MPL = w/p$ |
| “Lemons problem” | Check if $\mu \geq c_H$; if not, lemons equilibrium |
| “Adverse selection” | Asymmetric info → IC and IR constraints → separating/pooling |
| “Time inconsistency” | β - δ discounting; naive vs. sophisticated |
| “Second-order condition” | Compute Hessian; check NSD (max) or PSD (min) |
| “Envelope theorem” | $dV^*/d\alpha = \partial\mathcal{L}/\partial\alpha _{\text{optimum}}$ |
| “Expenditure function” | Dual to indirect utility; $e(p, v(p, m)) = m$ |

EXAM WARNING: Top Exam Traps

- Normalizing prices:** Set $p_y = 1$ early in GE problems — failure to normalize creates 2 unknowns from 1 equation.
- Leontief demands:** No interior tangency. Optimum at kink: $ax = by$, solve with budget.
- IFT sign:** Always check the denominator F_x first. If $F_x < 0$ (typical at max), then

$\partial x^*/\partial \alpha = -F_\alpha/F_x$ has sign = sign of F_α .

4. **Pigouvian tax level:** $t^* = D'(q^S)$, not $D'(q^P)$. Evaluate MED at the *socially* optimal quantity.
5. **Slutsky vs. Hicks:** Slutsky matrix uses Marshallian demands and is observable. Hicks substitution matrix uses Hicksian demands and is the theoretical NSD object.
6. **CARA MGF:** The trick is $E[e^{-\gamma \tilde{X}}] = e^{-\gamma \mu + \gamma^2 \sigma^2 / 2}$ — don't forget the 1/2.
7. **Robinson Crusoe equilibrium wage:** $w^* = p \cdot f'(L^*)$. Real wage = MPL .
8. **Bertrand paradox:** Two firms sufficient for $P = MC$ — only if goods are homogeneous and firms have equal costs.