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Econ 101A

Cumulative Final Practice Deck

A Neuroplastic Pedagogical Manual — Whole-Course Mastery

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How To Use This Deck (Read First). Each problem is engineered around five neuroplastic principles: (1) a *Memory Anchor* — one vivid metaphor that locks the technique into long-term memory; (2) a *Pattern Recognition Cue* — the trigger that tells you, at exam-time, which weapon to draw; (3) a *Full Derivation* from first principles with no skipped algebra; (4) an *Edge-Case Stress Test* that breaks the model on purpose so you learn its joints; (5) a *Cross-Link* back to other problems in this deck. Do problems *actively*: cover the solution, attempt fully, then unmask. Repeat any problem you cannot reproduce cold within 72 hours — this is when synaptic consolidation peaks.

Contents

Part I — Consumer Theory & Duality (MT1)

1 Problem 1 — Cobb-Douglas Demand & Slutsky Decomposition

Problem Statement

A consumer has utility $U(x, y) = x^{1/3}y^{2/3}$, faces prices (p_x, p_y) and income $m > 0$.

(A) Derive Marshallian demands $x^*(p_x, p_y, m)$ and $y^*(p_x, p_y, m)$ via the Lagrangian.

(B) Derive the indirect utility $V(p_x, p_y, m)$ and verify Roy's Identity.

(C) Derive the expenditure function $e(p_x, p_y, u)$ and Hicksian demand $h_x(p_x, p_y, u)$ via Shephard's Lemma.

(D) Decompose $\partial x^*/\partial p_x$ into substitution and income effects (Slutsky). Sign each.

(E) *Edge case:* What would change qualitatively if utility were $U = x^{1/3}y^{2/3} + 100$? Why?

Memory Anchor — "The Fixed-Share Wallet"

A Cobb-Douglas consumer carries a magic wallet with two compartments stitched in fixed proportions: fraction α for x, fraction $(1 - \alpha)$ for y. Prices and income may rise or fall, but the *stitching* never moves. Picture a literal leather wallet with one pocket sized α of the total — this image alone is enough to reconstruct $x^* = \alpha m/p_x$ at exam time.

Pattern Recognition Cue

Whenever you see $U = x^a y^b$ with $a, b > 0$ (Cobb-Douglas), *do not grind the Lagrangian unless asked.* Jump straight to the income-share rule:

$$x^* = \frac{a}{a+b} \cdot \frac{m}{p_x}, \qquad y^* = \frac{b}{a+b} \cdot \frac{m}{p_y}.$$

The Lagrangian is only needed for *showing work.* Master it once below, then short-circuit.

Part (A) — Marshallian Demand

Lagrangian:

$$\mathcal{L} = x^{1/3}y^{2/3} + \lambda(m - p_x x - p_y y).$$

FOCs:

$$\tfrac{1}{3}x^{-2/3}y^{2/3} = \lambda p_x,$$
$$\tfrac{2}{3}x^{1/3}y^{-1/3} = \lambda p_y.$$

MRS condition (divide):

$$\frac{1}{2}\cdot\frac{y}{x}=\frac{p_x}{p_y}\implies y=\frac{2p_x}{p_y}x.$$

Sub into budget $p_x x + p_y y = m$:

$$p_x x + p_y\cdot\tfrac{2p_x}{p_y}x = 3p_x x = m \implies \boxed{x^*(p_x,p_y,m)=\frac{m}{3p_x},\quad y^*(p_x,p_y,m)=\frac{2m}{3p_y}.}$$

Income-share check: $p_x x^* = m/3$ and $p_y y^* = 2m/3$. ✓ The wallet is stitched 1/3, 2/3.

Part (B) — Indirect Utility & Roy's Identity

Plug x^*, y^* into U:

$$V(p_x,p_y,m)=\left(\tfrac{m}{3p_x}\right)^{1/3}\left(\tfrac{2m}{3p_y}\right)^{2/3}=\frac{m}{3}\cdot\frac{2^{2/3}}{p_x^{1/3}p_y^{2/3}}.$$

Roy's Identity check $x^*=-\dfrac{\partial V/\partial p_x}{\partial V/\partial m}$:

$$\frac{\partial V}{\partial p_x}=-\tfrac{1}{3}\cdot\frac{m}{3}\cdot\frac{2^{2/3}}{p_x^{4/3}p_y^{2/3}},\qquad \frac{\partial V}{\partial m}=\frac{2^{2/3}}{3\,p_x^{1/3}p_y^{2/3}}.$$

$$-\frac{\partial V/\partial p_x}{\partial V/\partial m}=\frac{(1/3)(m/3)(2^{2/3})/(p_x^{4/3}p_y^{2/3})}{(2^{2/3})/(3p_x^{1/3}p_y^{2/3})}=\frac{m}{3p_x}=x^*.\ ✓$$

Part (C) — Expenditure Function & Hicksian Demand

Dual problem: $\min_{x,y} p_x x + p_y y$ s.t. $x^{1/3}y^{2/3}\ge u$.
The MRS condition is identical to the primal: $y=(2p_x/p_y)x$. Plug into constraint:

$$x^{1/3}\left(\tfrac{2p_x}{p_y}\right)^{2/3}x^{2/3}=u\implies x\cdot\left(\tfrac{2p_x}{p_y}\right)^{2/3}=u\implies \boxed{h_x(p_x,p_y,u)=u\left(\tfrac{p_y}{2p_x}\right)^{2/3}.}$$

Symmetrically: $h_y = u(2p_x/p_y)^{1/3}$.
Expenditure function:

$$e(p_x,p_y,u)=p_x h_x + p_y h_y = u\cdot p_x^{1/3}p_y^{2/3}\cdot\left[2^{-2/3}+2^{1/3}\right]=u\cdot p_x^{1/3}p_y^{2/3}\cdot\tfrac{3}{2^{2/3}}.$$

Shephard's Lemma check: $\partial e/\partial p_x = u\cdot\frac{1}{3}p_x^{-2/3}p_y^{2/3}\cdot 3/2^{2/3} = u(p_y/2p_x)^{2/3} = h_x$. ✓

Part (D) — Slutsky Decomposition for $\partial x^*/\partial p_x$

Slutsky equation:

$$\underbrace{\frac{\partial x^*}{\partial p_x}}_{\text{Total}}=\underbrace{\frac{\partial h_x}{\partial p_x}}_{\text{Substitution }(\le 0)}\underbrace{-x^*\cdot\frac{\partial x^*}{\partial m}}_{\text{Income}}.$$

Compute each piece (evaluate at $u = V(p_x, p_y, m)$ so Hicksian and Marshallian agree):

$$\frac{\partial x^*}{\partial p_x} = -\frac{m}{3p_x^2} \quad \text{(total, negative).}$$

$$\frac{\partial h_x}{\partial p_x} = -\tfrac{2}{3}\, u\, p_y^{2/3}\, 2^{-2/3}\, p_x^{-5/3} < 0 \quad \text{(substitution, strictly negative).}$$

$$x^* \cdot \tfrac{\partial x^*}{\partial m} = \tfrac{m}{3p_x} \cdot \tfrac{1}{3p_x} = \tfrac{m}{9p_x^2} \quad \text{(income piece, positive: } x \text{ is normal).}$$

Signs: Sub effect < 0, income effect $-x^* \partial x^* / \partial m < 0$. Both push the same direction: total demand falls. Cobb-Douglas goods are always normal — no Giffen, no inferior.

Part (E) — Edge Case: Adding a Constant

$U + 100$ is a *monotone transformation* (in fact, an affine shift). Preferences are *ordinal*: any strictly increasing transformation of U leaves the ranking of bundles intact. $\Rightarrow$ Marshallian demands, indirect utility (up to its level), Hicksian demands, and expenditure are *unchanged* as functions of (p, m). Only the cardinal value of utility shifts. This is why we never report "how much utility" — we only report demands and rankings.

Cross-Link

The duality identities (Roy, Shephard) seen here are reused verbatim in **Problem 5** (producer cost minimization). Producer Shephard $\Leftrightarrow$ Consumer Shephard — same math, mirrored variables. The "mirror dimension" you build now powers half of MT2.

2 Problem 2 — Quasilinear Preferences & Pure Substitution

Problem Statement

$U(x,y) = \ln(x) + y$, prices (p_x, p_y), income m. Assume m large enough that $y^* > 0$.

(A) Derive Marshallian demands. Why is x^* independent of m?

(B) Compute the income effect $\partial x^*/\partial m$. What does this tell you about the Slutsky decomposition?

(C) Find the Hicksian demand $h_x(p_x, p_y, u)$ and verify it equals Marshallian at the right u.

(D) Compute consumer surplus from a price drop $p_x : 2 \to 1$ (with $p_y = 1$).

Memory Anchor — "The Necessity Bucket"

Quasilinear utility = $\ln(x)$ is a *bucket with a fixed lid.* Once the bucket is full (MU equals price ratio), every additional dollar of income spills past it into y, the linear "money-equivalent" good. Picture a bucket with a literal lid: more rain (income) does not raise its level; the rain runs off into a swimming pool (y).

Pattern Recognition Cue

$U = v(x) + y$ with v concave $\Rightarrow$ **No income effect on x.** The Slutsky equation collapses to total = substitution. This is *the* reason quasilinear is the workhorse of public-finance and welfare economics: consumer surplus is exact, not an approximation.

Parts (A)–(B)

FOCs: $1/x = \lambda p_x,\ 1 = \lambda p_y$. Hence $\lambda = 1/p_y$ and

$$\boxed{x^*(p_x, p_y) = \frac{p_y}{p_x}, \qquad y^*(p_x, p_y, m) = \frac{m - p_y}{p_y} = \frac{m}{p_y} - 1.}$$

$\partial x^*/\partial m = 0$. The Engel curve for x is vertical: a Veblen-free, fashion-free, plain "need."

Part (C) — Hicksian

Dual: $\min p_x x + p_y y$ s.t. $\ln x + y \geq u$. Tangency: $1/x = p_x/p_y \Rightarrow x = p_y/p_x$. Plug into constraint: $\ln(p_y/p_x) + y = u \Rightarrow y = u - \ln(p_y/p_x)$.

$$\boxed{h_x(p_x, p_y, u) = \frac{p_y}{p_x} = x^*.}$$

Marshallian = Hicksian *everywhere*. This is the algebraic tattoo of "no income effect."

Part (D) — Exact Consumer Surplus

With quasilinear utility, $\Delta CS = \int_{p_1}^{p_0} x^*(p)\, dp$ is *exact* (not an approximation). With $p_y = 1$:

$$\Delta CS = \int_1^2 \frac{1}{p_x}\, dp_x = \ln 2 \approx 0.693.$$

The consumer is exactly $\ln 2$ utils (and dollars, since y is the numeraire) better off.

Edge Case — The Corner

If $m < p_y$, the formula gives $y^* < 0$, which is infeasible. The consumer corners at $y^* = 0$, $x^* = m/p_x$. **Always check the corner before reporting.** An exam love trick: set m small.

Cross-Link

The "no income effect" property here is exactly what makes **Problem 11** (Pigouvian taxation) clean: with quasilinear preferences, the deadweight-loss triangle is the literal welfare loss, not an approximation. The integrals are honest.

3 Problem 3 — Intertemporal Choice & IFT Comparative Statics

Problem Statement

A consumer lives two periods. Utility $U(c_0, c_1) = \ln c_0 + \beta \ln c_1$, with discount factor $\beta \in (0, 1)$. Endowed income y_0 today, y_1 tomorrow. Saving s earns gross return $R = 1 + r$. Budget: $c_0 = y_0 - s$, $c_1 = y_1 + Rs$.

(A) Derive optimal saving $s^*(R, \beta, y_0, y_1)$.

(B) Use the Implicit Function Theorem on the Euler equation to sign $\partial s^*/\partial R$ *without* solving explicitly. Identify substitution vs. income effects.

(C) Edge case: under what condition on (y_0, y_1, β, R) does the consumer borrow ($s^* < 0$)?

Memory Anchor — "The Patience Tug-of-War"

The Euler equation $u'(c_0) = \beta R\, u'(c_1)$ is two hands tugging on a single rope. Left hand: *impatience* (you want to consume now). Right hand: *return* (the bank multiplies tomorrow's dollars by R, weighted by your patience β). The optimal allocation is the rope-knot's resting position. Raising R pulls the right hand harder; raising β pulls it harder too. They are mathematically symmetric: *patience and interest are interchangeable.*

Pattern Recognition Cue

Two-period consumption ⇒ **Euler equation** $u'(c_0) = \beta R\, u'(c_1)$. For log utility this becomes $1/c_0 = \beta R/c_1$, or $c_1 = \beta R\, c_0$. Combined with the *intertemporal budget constraint* $c_0 + c_1/R = y_0 + y_1/R \equiv W$ (lifetime wealth), you solve linearly.

Part (A) — Optimal Saving

Lifetime wealth $W = y_0 + y_1/R$. Euler ⇒ $c_1 = \beta R\, c_0$. Plug into budget:

$$c_0 + \beta c_0 = W \Rightarrow c_0^* = \frac{W}{1+\beta}, \qquad c_1^* = \frac{\beta R W}{1+\beta}.$$

$$\boxed{s^* = y_0 - c_0^* = y_0 - \frac{1}{1+\beta}\left(y_0 + \frac{y_1}{R}\right) = \frac{\beta y_0 - y_1/R}{1+\beta}.}$$

Part (B) — IFT on the Euler Equation

Let $F(s, R) \equiv u'(y_0 - s) - \beta R\, u'(y_1 + Rs) = 0$ implicitly define $s^*(R)$.

IFT: $\dfrac{\partial s^*}{\partial R} = -\dfrac{\partial F/\partial R}{\partial F/\partial s}$.

Numerator $\partial F/\partial R$:

$$-\beta u'(c_1) - \beta R\, u''(c_1) \cdot s.$$

Denominator $\partial F/\partial s$:

$$-u''(c_0) - \beta R^2\, u''(c_1) > 0 \quad \text{(since } u'' < 0\text{, both terms become positive).}$$

Hence

$$\boxed{\frac{\partial s^*}{\partial R} = \frac{\beta u'(c_1) + \beta R\, u''(c_1)\, s}{-u''(c_0) - \beta R^2 u''(c_1)} = \underbrace{\frac{\beta u'(c_1)}{D}}_{\text{Substitution} > 0} + \underbrace{\frac{\beta R u''(c_1)\, s}{D}}_{\text{Income, sign}=-\operatorname{sgn}(s)}.}$$

Reading the signs: substitution effect > 0 (higher R rewards saving). Income effect's sign *depends on whether the consumer is a saver or a borrower.* If $s > 0$ (saver), the income effect is *negative* (richer in PV terms ⇒ consume more today ⇒ save less). For a borrower ($s < 0$), it reinforces substitution. *This is why Powell-style exams love this problem: the sign of $\partial s^*/\partial R$ is genuinely ambiguous for savers.*

Part (C) — Borrowing Threshold

$s^* < 0 \iff \beta R\, y_0 < y_1 \iff y_1/y_0 > \beta R$. Intuition: if tomorrow's income is large enough relative to today's (scaled by patience and return), the consumer pulls forward consumption by borrowing. This is the canonical *young grad student* case — low y_0, high expected y_1.

Cross-Link

The IFT machinery here is identical to **Problem 7** (tax incidence). Both rely on: 1) write FOC as implicit function $F = 0$; 2) differentiate; 3) sign denominator via SOC, sign numerator economically. Master IFT once, deploy it forever.

4 Problem 4 — Labor-Leisure Choice with a Tax

Problem Statement

Worker has utility $U(c,\ell) = \ln c - \ell^2/2$ where $\ell \in [0,T]$ is hours worked. Wage w, after-tax wage $(1-\tau)w$, no non-labor income. Budget: $c = (1-\tau)w\ell$.

(A) Derive optimal labor supply ℓ^*.

(B) Sign $\partial\ell^*/\partial\tau$. Does an income-tax cut increase work?

(C) How does the answer change if utility is $U = \ln c + \ln(T-\ell)$ (logarithmic leisure)?

Memory Anchor — "The Rubber Band"

Labor supply has two forces: a *rubber band* pulling toward more leisure (disutility $\ell^2/2$ snaps back harder the more you work) and a *magnet* pulling toward consumption (higher w pulls harder). Taxes *loosen the magnet.* Whether you work more or less depends on which force dominates — and the form of utility decides.

Part (A)

Substitute budget into U: $\max_\ell \ln[(1-\tau)w\ell] - \ell^2/2$.

FOC: $1/\ell - \ell = 0 \Rightarrow \ell^* = 1$. *Independent of τ and w.*

Why? Log substitution from $\ln[(1-\tau)w\ell] = \ln(1-\tau) + \ln w + \ln\ell$. Wages shift the constant, not the marginal trade-off. *Income and substitution effects exactly cancel.*

Part (B)–(C)

(B) $\partial\ell^*/\partial\tau = 0$. Tax cut yields no labor response under this utility. This is the *log-quasi-linear miracle.*

(C) With $U = \ln c + \ln(T-\ell)$:

$$\max_\ell \ \ln[(1-\tau)w\ell] + \ln(T-\ell).$$

FOC: $1/\ell - 1/(T-\ell) = 0 \Rightarrow \ell^* = T/2$. *Still independent of τ, w.* The unit Cobb-Douglas split applies again, now in time-shares: half-and-half.

Different utility, different answer: CES leisure $U = \ln c - \ell^{1+1/\eta}/(1+1/\eta)$. Then FOC gives $\ell^* = (1-\tau)^\eta w^\eta \cdot (\text{const})$. Here $\partial\ell^*/\partial\tau < 0$ (taxes reduce work) when $\eta > 0$. The Frisch elasticity η is precisely the *percent change in labor per percent change in $(1-\tau)w$.*

Edge Case — The Backward-Bending Supply Curve

For utilities like $U = c - \ell^2/2$ (linear consumption), labor supply is $\ell^* = w(1-\tau)$. At low wages, raising w raises labor (substitution dominates). At very high wages with sufficient income effect (e.g., adding non-labor income), labor supply can *bend backward.* UC Berkeley loves to put a "tipping wage" in problem sets — always check whether the income effect can swamp.

Cross-Link

This problem trains the same algebra as **Problem 3** (intertemporal). Conceptually, *leisure* and *tomorrow's consumption* both serve as "the alternative" to work-now or consume-now. Master one, the other comes free.

Part II — Producer Theory & Markets (MT2)

5 Problem 5 — Cost Minimization & Conditional Factor Demand

Problem Statement

A firm has production function $f(K,L) = K^{1/2}L^{1/2}$ (Cobb-Douglas, CRS), input prices r and w, target output y.

(A) Derive conditional factor demands $K^*(w,r,y)$ and $L^*(w,r,y)$.
(B) Derive the cost function $c(w,r,y)$ and verify Shephard's Lemma: $\partial c/\partial w = L^*$.
(C) Derive marginal and average cost. Show $MC = AC$ at every y (CRS hallmark).
(D) Edge case: what changes if the production function becomes $f = K^{1/2}L^{1/2} - F$ with $F > 0$ a fixed cost? Plot the resulting AC.

Memory Anchor — "The Mirror Dimension"

Cost minimization is the consumer problem reflected through a mirror: "utility" becomes "output," "income" becomes "cost," Marshallian becomes *conditional factor demand*, and the indirect utility V becomes the cost function c. Every theorem you learned for consumers has a producer twin: Roy's identity ↔ Hotelling's Lemma; Shephard's lemma is literally the same theorem.

Pattern Recognition Cue

Cobb-Douglas production with exponents α, β and CRS ($\alpha + \beta = 1$) $\Rightarrow$ $c(w,r,y) = A\,w^{\alpha}r^{\beta}\,y^{1/(\alpha+\beta)}$ with A a constant. The exponent on y *is* the inverse degree of homogeneity: tells you returns to scale at a glance.

Parts (A)–(B)

Lagrangian: $\mathcal{L} = wL + rK + \mu(y - K^{1/2}L^{1/2})$. FOCs:

$$w = \mu \cdot \tfrac{1}{2}K^{1/2}L^{-1/2}, \qquad r = \mu \cdot \tfrac{1}{2}K^{-1/2}L^{1/2}.$$

Ratio: $w/r = L/K \Rightarrow K = (w/r)L$. Plug into constraint:

$$y = [(w/r)L]^{1/2}L^{1/2} = L(w/r)^{1/2} \Rightarrow L^* = y(r/w)^{1/2}, \quad K^* = y(w/r)^{1/2}.$$

Cost:

$$\boxed{c(w,r,y) = wL^* + rK^* = 2y\sqrt{wr}.}$$

Shephard check: $\partial c/\partial w = y\sqrt{r/w} = L^*$. ✓

Part (C)

$MC = \partial c/\partial y = 2\sqrt{wr}$. $AC = c/y = 2\sqrt{wr}$. $MC = AC$ everywhere. This is the *visual signature* of CRS: a horizontal cost curve.

Part (D) — Fixed Cost Breaks Homogeneity

With a fixed cost, $c(y) = 2y\sqrt{wr} + F$. Then $AC = 2\sqrt{wr} + F/y$, which decreases without bound as $y \to \infty$ (*economies of scale*) and explodes near $y = 0$. $MC = 2\sqrt{wr}$ stays flat. The U-shape so beloved on exams arises only when MC itself slopes up (decreasing returns or quadratic cost).

Cross-Link

Conditional factor demand (here) plus output choice (Problem 6) is the full producer story. In **Problem 13** (Robinson Crusoe) we glue this to consumer theory: the consumer-firm earns profits and spends them on consumption.

6 Problem 6 — Profit Maximization & the Hessian SOC

Problem Statement

A price-taking firm has production $f(K, L) = K^\alpha L^\beta$ with $\alpha, \beta > 0$ and $\alpha + \beta < 1$ (decreasing returns). Output price p, factor prices w, r.

(A) Set up unconstrained profit max in (K, L). Derive FOCs.
(B) Verify SOC by signing the Hessian of profit. State the determinant condition.
(C) Solve for K^* and L^* as functions of (p, w, r).
(D) Why does $\alpha + \beta < 1$ matter? What goes wrong if $\alpha + \beta = 1$ (CRS)?

Memory Anchor — "The Bowl That Holds Water"

A maximum exists only if profit is a *bowl turned upside-down* (concave). The Hessian's "shape" must be *negative definite*: bowl up = no max; bowl down = unique max. With CRS, the bowl flattens into a plane — *infinitely many* optima, none of them strict. With IRS, the bowl inverts: profit explodes to $+\infty$. The SOC is your structural integrity test.

Pattern Recognition Cue

Two-input profit max $\Rightarrow$ 2×2 Hessian. NSD requires:

$$H_{11} < 0, \quad H_{22} < 0, \quad \det H = H_{11}H_{22} - H_{12}^2 > 0.$$

The cross-partial determinant condition is what "kills" CRS: at $\alpha + \beta = 1$, $\det H = 0$.

Parts (A)–(B)

Profit: $\pi(K, L) = pK^\alpha L^\beta - rK - wL$.

$$\pi_K = p\alpha K^{\alpha-1} L^\beta - r = 0,$$
$$\pi_L = p\beta K^\alpha L^{\beta-1} - w = 0.$$

Hessian:

$$H = p \begin{pmatrix} \alpha(\alpha-1)K^{\alpha-2}L^\beta & \alpha\beta K^{\alpha-1}L^{\beta-1} \\ \alpha\beta K^{\alpha-1}L^{\beta-1} & \beta(\beta-1)K^\alpha L^{\beta-2} \end{pmatrix}.$$

$H_{11}, H_{22} < 0$ since $\alpha, \beta < 1$. Determinant:

$$\det H = p^2\alpha\beta K^{2\alpha-2}L^{2\beta-2}\big[(\alpha-1)(\beta-1) - \alpha\beta\big] = p^2\alpha\beta K^{2\alpha-2}L^{2\beta-2}(1-\alpha-\beta).$$

$\det H > 0 \iff \alpha + \beta < 1$. SOC holds strictly only under DRS. ✓

Part (C) — Closed-Form Optimum

Divide FOCs: $\dfrac{\alpha L}{\beta K} = \dfrac{r}{w} \Rightarrow K = \dfrac{\alpha w}{\beta r} L$. Sub into FOC for L:

$$p\beta \left(\tfrac{\alpha w}{\beta r}\right)^\alpha L^{\alpha+\beta-1} = w \Rightarrow L^* = \left(\tfrac{p\beta}{w}\right)^{\frac{1-\alpha}{1-\alpha-\beta}} \left(\tfrac{p\alpha}{r}\right)^{\frac{\beta}{1-\alpha-\beta}}.$$

K^* symmetric. The exponents $1/(1-\alpha-\beta)$ are the source of the explosive supply elasticities you see in Berkeley exams.

Part (D) — The CRS Trap

At $\alpha + \beta = 1$: $\det H = 0$. Hessian is negative *semi*-definite, not negative definite. Profit is constant along rays $(K, L) = t(K_0, L_0)$. *No unique optimum.* If $p \cdot \text{MPK}(K_0, L_0) > r$ at the FOC ratio, scale to infinity; if less, shut down. Hence in **competitive equilibrium with CRS, profits are exactly zero** — because nothing else is finite.

Cross-Link

The CRS knife-edge here is exactly why **Problem 13** (Robinson Crusoe with CRS) requires zero-profit pricing $p = MC$. The micro-meso-macro chain emerges: SOCs decide which assumption the model leans on.

7 Problem 7 — Tax Incidence via the Implicit Function Theorem

Problem Statement

Market demand $D(p)$ with $D' < 0$. Market supply by competitive firms $S(p)$ with $S' > 0$. Government imposes a per-unit tax t on producers, so producers receive $p - t$ at consumer price p. Equilibrium: $D(p^*) = S(p^* - t)$.

(A) Use IFT to derive $\dfrac{\partial p^*}{\partial t}$.

(B) Show that $0 \leq \partial p^*/\partial t \leq 1$ generically, and identify the limits.

(C) Express *tax incidence* (consumer share of t) in terms of demand and supply elasticities $\varepsilon_D, \varepsilon_S$.

Memory Anchor — "The Inelastic Sucker"

Tax incidence is a tug-of-war between two ropes (demand and supply). Whichever rope is *stiffer* (more inelastic) takes the bigger share of the tax. "Stiffer = stuck = stuck with the bill." Picture two arm-wrestlers; the one who can't bend their arm absorbs the impact.

Pattern Recognition Cue

The IFT recipe in three lines: **(1)** write FOC/equilibrium as $F(\text{endogenous}, \text{parameter}) = 0$; **(2)** $\partial \text{endog}/\partial \text{param} = -F_{\text{param}}/F_{\text{endog}}$; **(3)** sign denominator via SOC/stability, sign numerator via economics.

Part (A) — IFT Derivation

Let $F(p, t) \equiv D(p) - S(p - t) = 0$.

$$F_p = D'(p) - S'(p - t), \qquad F_t = S'(p - t).$$

$$\boxed{\frac{\partial p^*}{\partial t} = -\frac{F_t}{F_p} = -\frac{S'}{D' - S'} = \frac{S'}{S' - D'}}.$$

Since $S' > 0$ and $D' < 0$, denominator $S' - D' > 0$, so $\partial p^*/\partial t \in (0, 1)$.

Part (B)–(C) — Limits and Elasticity Form

Multiply numerator and denominator by p/Q:

$$\frac{\partial p^*}{\partial t} = \frac{S' \cdot p/Q}{(S' - D') \cdot p/Q} = \frac{\varepsilon_S}{\varepsilon_S + |\varepsilon_D|}.$$

Limits:

- $\varepsilon_D \to 0$ (perfectly inelastic demand): $\partial p^*/\partial t \to 1$ — consumers eat 100% of the tax.
- $\varepsilon_S \to 0$: $\partial p^*/\partial t \to 0$ — producers eat 100%.
- Equal elasticities: 50/50 split.

Famous corollary: who *statutorily* bears the tax (consumer side vs producer side) is irrelevant in equilibrium — only elasticities matter. This is the cleanest result in public finance.

Edge Case — Specific vs Ad Valorem

Replace per-unit t with ad valorem τ (proportional). Equilibrium: $D(p) = S(p(1-\tau))$. The IFT machine still runs; only the chain rule changes ($S' \cdot p$ instead of $S' \cdot 1$). The elasticity formula is identical. *Tax form is a labeling choice*, not a real difference.

Cross-Link

The same elasticity-share logic governs **Problem 8** (the inverse-elasticity rule for price discrimination): high elasticity = low markup, low elasticity = high markup. Same physics, different clothes.

8 Problem 8 — Monopoly & Third-Degree Price Discrimination

Problem Statement

A monopolist faces two segmented markets with demands $Q_1(p_1)$ and $Q_2(p_2)$, constant marginal cost c. It can charge different prices.

(A) Derive the inverse-elasticity rule: $(p_i - c)/p_i = 1/|\varepsilon_i|$.

(B) In a single-market monopoly with linear demand $p = a - bQ$ and MC c, compute the deadweight loss (DWL).

(C) Suppose market 1 has $|\varepsilon_1| = 2$ and market 2 has $|\varepsilon_2| = 4$. Which market gets the higher price? By how much (ratio)?

Memory Anchor — "The Bicycle Lock"

A monopolist with two markets is a bicycle lock with two dials. Each dial sets one market's price. Spin each dial *independently* until $MR_i = MC$ in that market. The mathematical lock-pick: $(p - c)/p = 1/|\varepsilon|$. **Less elastic = higher markup.** Whisper it: *"inelastic gets gouged."*

Pattern Recognition Cue

Whenever you see "segmented markets" or "different consumer types" ⇒ apply the inverse-elasticity rule *per market*. Whenever you see "social welfare with monopoly" ⇒ compute DWL $= \frac{1}{2}(p^M - c)(Q^* - Q^M)$.

Part (A) — Inverse-Elasticity Rule

$\pi_i = p_i Q_i(p_i) - cQ_i(p_i)$. FOC:

$$Q_i + p_i Q_i' - cQ_i' = 0 \Rightarrow Q_i = (c - p_i)Q_i' = -(p_i - c)Q_i'.$$

Divide by Q_i' then by p_i:

$$\frac{Q_i}{p_i Q_i'} = -\frac{p_i - c}{p_i} \cdot 1 \Rightarrow \frac{p_i - c}{p_i} = -\frac{Q_i}{p_i Q_i'} = \frac{1}{|\varepsilon_i|}.$$

Markup is the reciprocal of demand elasticity, market by market.

Part (B) — DWL with Linear Demand

$p = a - bQ$. $MR = a - 2bQ = c \Rightarrow Q^M = (a - c)/(2b)$, $p^M = (a + c)/2$. Competitive: $p^* = c$, $Q^* = (a - c)/b$.

$$\text{DWL} = \tfrac{1}{2}(p^M - c)(Q^* - Q^M) = \tfrac{1}{2} \cdot \tfrac{a-c}{2} \cdot \tfrac{a-c}{2b} = \frac{(a-c)^2}{8b}.$$

Monopoly profit at the same parameters: $(p^M - c)Q^M = (a - c)^2/(4b)$. *DWL is exactly half of monopoly profit when demand is linear.* Memorize this ratio.

Part (C) — Numerical Application

$(p_1 - c)/p_1 = 1/2 \Rightarrow p_1 = 2c$.
$(p_2 - c)/p_2 = 1/4 \Rightarrow p_2 = (4/3)c \approx 1.33c$.
Ratio $p_1/p_2 = 3/2$. The less elastic market pays 50% more. Inelastic gets gouged. ✓

Edge Case — Pareto Improvement?

3rd-degree PD lowers the price for the elastic group (often the poor, who substitute), and raises it for the inelastic group (often the rich or the captured). Welfare *can* increase if newly served markets open up. The classic counter-example: textbooks priced lower in developing countries.

Cross-Link

The inverse-elasticity intuition reappears in optimal Ramsey taxation, in 1st-degree PD (where the markup formula collapses because consumer surplus is captured), and in welfare analysis of **Problem 11** (Pigouvian tax) where the optimal tax targets the social MC.

Part III — General Equilibrium, Risk & Synthesis (Final)

9 Problem 9 — General Equilibrium in an Edgeworth Box

Problem Statement

Two consumers A, B, two goods x, y. Endowments: $\omega_A = (2, 0)$, $\omega_B = (0, 2)$. Utilities: $U_A = x_A y_A$, $U_B = x_B y_B$. Normalize $p_y = 1$, let $p = p_x$.

(A) Derive each consumer's Marshallian demand as a function of p and their endowed wealth.
(B) Find the Walrasian equilibrium price p^* and allocation.
(C) Verify Walras' Law: market y also clears.
(D) Show the equilibrium is Pareto efficient by computing $MRS_A = MRS_B$.

Memory Anchor — "The Auctioneer's Coin"

General equilibrium is a Walrasian auctioneer juggling *one coin* (p_x/p_y). She tosses it higher when x is in excess demand and lower when in excess supply. The contract curve is the spine of all Pareto-efficient allocations; the equilibrium is the single point where the budget line *through both endowments* kisses the spine.

Pattern Recognition Cue

Pure-exchange GE with Cobb-Douglas: **(1)** wealth $= p\omega_x + \omega_y$; **(2)** demand for x is fraction-of-wealth divided by p_x; **(3)** sum demands, set = total endowment of x; **(4)** solve for p. Use Walras' Law to skip the second market.

Parts (A)–(B)

$m_A = 2p$, $m_B = 2$. Cobb-Douglas $\Rightarrow$ each spends half on each good:

$$x_A^* = \frac{m_A}{2p} = 1, \qquad y_A^* = \frac{m_A}{2} = p,$$

$$x_B^* = \frac{m_B}{2p} = \frac{1}{p}, \qquad y_B^* = \frac{m_B}{2} = 1.$$

Market clear x: $1 + 1/p = 2 \Rightarrow p^* = 1$. Allocation: $A = (1, 1)$, $B = (1, 1)$. *Equal split, equal price.*

Parts (C)–(D)

Market y: $p^* + 1 = 2$. ✓Walras' Law works as advertised.
$MRS_A = y_A/x_A = 1/1 = 1$. $MRS_B = y_B/x_B = 1/1 = 1$. $MRS_A = MRS_B = p^*/p_y = 1$.

✓This equality is the definition of efficiency in pure exchange: *no further trade possible.*

Edge Case — Asymmetric Endowment

If $\omega_A = (2, 0)$ but $U_A = x_A^{0.9} y_A^{0.1}$ (A loves x): A keeps almost all of x, p collapses (since A barely sells x, x becomes scarce in the market), B ends up with little. *Endowment + preferences jointly determine wealth* — a Berkeley exam favorite. The **First Welfare Theorem** still holds: equilibrium is efficient, even if unequal.

Cross-Link

This problem is the warm-up for **Problem 13** (Robinson Crusoe), where one of the agents is replaced by a firm. Same algebra, with profits flowing into income.

10 Problem 10 — Expected Utility, Risk Aversion, Certainty Equivalent

Problem Statement

Investor has utility $U(w) = \sqrt{w}$. Initial wealth $W_0 = 100$. A gamble pays $+44$ with probability $1/2$ and -19 with probability $1/2$.

(A) Compute expected wealth and expected utility of the gamble.
(B) Find the certainty equivalent (CE) and the risk premium ($\pi = E[w] - CE$).
(C) Compute the Arrow-Pratt coefficient of absolute risk aversion (ARA) at W_0.
(D) Edge case: if the same gamble were offered at $W_0 = 10000$, would the investor accept? Explain via decreasing absolute risk aversion (DARA).

Memory Anchor — "The Concave Hill"

Risk aversion is the geometry of a *concave hill.* The expected payoff lies on the chord connecting the two outcomes; the expected *utility* lies under the chord, on the curve. The vertical gap is the risk premium. Picture a literal hill: the chord is a tightrope between two cliffs; the curve is the ridge below it. Walking the ridge is always lower than the rope — because the hill is curved.

Pattern Recognition Cue

"Expected utility" problems decompose into 3 numbers: **(1)** $E[w]$ — the gamble's mean. **(2)** $E[U(w)]$ — weighted utility. **(3)** CE — the wealth w^* such that $U(w^*) = E[U(w)]$. The risk premium is $E[w] - CE$, always ≥ 0 for risk-averse agents.

Parts (A)–(B)

$E[w] = \frac{1}{2}(144) + \frac{1}{2}(81) = 112.5$. $E[U] = \frac{1}{2}\sqrt{144} + \frac{1}{2}\sqrt{81} = \frac{1}{2}(12) + \frac{1}{2}(9) = 10.5$. $CE : \sqrt{CE} = 10.5 \Rightarrow CE = 110.25$. Risk premium: $\pi = 112.5 - 110.25 = 2.25$. **Interpretation:** the investor would pay up to \$2.25 to avoid the risk altogether.

Part (C) — Arrow-Pratt

$U' = \frac{1}{2}w^{-1/2}$; $U'' = -\frac{1}{4}w^{-3/2}$.

$$ARA(w) = -\frac{U''}{U'} = -\frac{-w^{-3/2}/4}{w^{-1/2}/2} = \frac{1}{2w}.$$

At $w = 100$: $ARA = 1/200 = 0.005$. ARA *decreases* with wealth: square-root utility exhibits **DARA** — richer agents are absolutely less risk-averse.

Part (D) — Wealth Effect

At $W_0 = 10000$, the same $\pm$ shocks $(+44, -19)$ are tiny relative to wealth. $E[U] = \frac{1}{2}\sqrt{10044} + \frac{1}{2}\sqrt{9981} \approx \frac{1}{2}(100.220) + \frac{1}{2}(99.905) = 100.0625$. $U(W_0) = 100$. Since $E[U] > U(W_0)$, accept. The DARA property predicts this: as $w \to \infty$, $ARA \to 0$, and any actuarially favorable bet $(E[w] > W_0)$ is eventually accepted. *Wealth buys risk-tolerance.*

CRRA vs CARA — The Two Workhorses

CRRA $U = w^{1-\rho}/(1-\rho)$: relative risk aversion ρ constant. Risky-asset share independent of wealth. Used in macro/finance. **CARA** $U = -e^{-\gamma w}$: absolute risk aversion γ constant. Dollar amount in risky asset independent of wealth. Used in info economics. Berkeley loves CARA + Normal payoffs because they yield closed-form CE: $CE = E[w] - \frac{\gamma}{2}\text{Var}(w)$. **Memorize this formula.**

Cross-Link

The CARA-Normal CE formula is the engine of the existing `Final_Exam_Prep_Solutions.tex` Problem 2. If you can derive that CE formula from the moment-generating function, you have mastered both this problem and that one.

11 Problem 11 — Externalities & Pigouvian Taxation

Problem Statement

A factory produces output q at private cost $C(q) = q^2/2$, sells at price $p = 10$. Each unit emits pollution generating external damage $D(q) = q^2$ on a downstream neighbor.

(A) Find the private (laissez-faire) output q^P.
(B) Find the social optimum q^S.
(C) Find the Pigouvian per-unit tax t^* that implements q^S.
(D) Compute the deadweight loss of laissez-faire (the welfare gap).

Memory Anchor — "The Phantom Cost"

An externality is a cost the firm doesn't see — a phantom rider on every unit. The Pigouvian tax makes the phantom *visible*: it sets t = marginal external damage at the optimum, so the firm's private FOC *coincides* with the social FOC. *Make the invisible cost visible.*

Pattern Recognition Cue

Externality problems are always: **(i)** solve private FOC ($p = MC_{\text{private}}$); **(ii)** solve social FOC ($p = MC_{\text{private}} + MC_{\text{external}}$); **(iii)** Pigouvian $t^* = MC_{\text{external}}(q^S)$.

Solutions (A)–(D)

Private: $\max pq - q^2/2 \Rightarrow q^P = p = 10$.
Social: $\max pq - q^2/2 - q^2 \Rightarrow p - q - 2q = 0 \Rightarrow q^S = p/3 = 10/3$.
Pigouvian tax: The firm's tax-augmented FOC is $p - q - t = 0 \Rightarrow q = p - t$. Set this equal to $q^S = 10/3$: $t^* = p - q^S = 10 - 10/3 = 20/3 \approx 6.67$. **Verification:** $t^* = MD(q^S) = 2q^S = 20/3$. ✓
DWL of laissez-faire: the welfare loss is the integral of $(MSC - MB)$ from q^S to q^P:

$$\text{DWL} = \int_{q^S}^{q^P} [(q + 2q) - p]\, dq = \int_{10/3}^{10} (3q - 10)\, dq.$$

Compute: $\left[\frac{3q^2}{2} - 10q\right]_{10/3}^{10} = (150 - 100) - (\frac{3\cdot 100/9}{2} - \frac{100}{3}) = 50 - (\frac{50}{3} - \frac{100}{3}) = 50 + \frac{50}{3} = \frac{200}{3} \approx 66.67$.

Edge Case — Subsidies, Bargaining, Coase

If the externality is positive (e.g., a vaccine), the same algebra gives a Pigouvian *subsidy*: $s^* > 0$ pays the firm to expand. **Coase Theorem:** with zero transaction costs and well-defined property rights, parties bargain to q^S *without* a tax. Pigou and Coase are *complementary*, not contradictory — they describe the same target via different instruments.

Cross-Link

The DWL geometry here is identical to the monopoly DWL of **Problem 8**. Both compute the triangular gap between the social-optimal quantity and the privately-chosen quantity. Different distortion, same triangle.

12 Problem 12 — Cournot Duopoly Nash Equilibrium

Problem Statement

Two firms produce a homogeneous good. Inverse demand: $P = a - b(q_1 + q_2)$. Identical constant marginal cost $c < a$.

(A) Derive each firm's best response $q_i(q_j)$.
(B) Solve for the symmetric Cournot-Nash equilibrium (q_1^*, q_2^*) and price P^*.
(C) Compare profits to monopoly and to perfect competition. Which lies in the middle?
(D) Show that as the number of identical firms $n \to \infty$, the equilibrium converges to perfect competition.

Memory Anchor — "Two Wolves Sharing a Carcass"

Cournot duopolists are two wolves circling the same carcass. Each wolf chooses how much to eat, *taking the other's bite as fixed*. The Nash equilibrium is the bite-pair where neither wolf wants to bite more or less, given the other's bite. The carcass shrinks (price falls) the more they both eat — so they both eat less than a single wolf (monopolist) would, but more than a perfectly competitive pack.

Pattern Recognition Cue

Cournot recipe: **(1)** write $\pi_i = (P - c)q_i$ with $P = a - b(q_i + q_j)$; **(2)** FOC in q_i holding q_j fixed gives best-response line; **(3)** solve simultaneously by symmetry. With n symmetric firms, $q_i^* = (a - c)/[(n + 1)b]$. The factor $1/(n + 1)$ is the Cournot signature.

Parts (A)–(B)

$\pi_i = [a - b(q_i + q_j) - c]q_i$.

FOC: $a - 2bq_i - bq_j - c = 0 \Rightarrow q_i(q_j) = \dfrac{a - c - bq_j}{2b}$.

By symmetry $q_1^* = q_2^* = q^*$:

$$q^* = \frac{a - c - bq^*}{2b} \Rightarrow 2bq^* + bq^* = a - c \Rightarrow q^* = \frac{a - c}{3b}.$$

Total $Q^* = 2(a - c)/(3b)$. Price $P^* = a - bQ^* = (a + 2c)/3$.
Profits per firm: $(P^* - c)q^* = \frac{a-c}{3} \cdot \frac{a-c}{3b} = (a - c)^2/(9b)$.

Parts (C)–(D)

Comparison table:

Structure	Q	P	Industry profit
Perfect comp.	$(a - c)/b$	c	0
Cournot duopoly	$2(a - c)/(3b)$	$(a + 2c)/3$	$2(a - c)^2/(9b)$
Monopoly	$(a - c)/(2b)$	$(a + c)/2$	$(a - c)^2/(4b)$

Cournot lies strictly between perfect competition and monopoly. ✓
n-firm generalization: $q_i^* = (a-c)/[(n+1)b]$, $Q^* = n(a-c)/[(n+1)b] \to (a-c)/b$ as $n \to \infty$. $P^* \to c$, profit per firm $\to 0$. *Cournot converges to PC.*

Edge Case — Stackelberg Asymmetry

If firm 1 moves first (Stackelberg), it internalizes firm 2's best response: $q_1 = (a-c)/(2b)$, $q_2 = (a-c)/(4b)$. Total $Q = 3(a-c)/(4b)$, between Cournot and PC. The *first-mover advantage* is real: leader profit > Cournot profit, follower profit < Cournot profit. Commitment is power.

Cross-Link

The convergence-to-PC limit ($n \to \infty$) is the same intuition that justifies treating **Problem 7**'s competitive supply as the limit of many firms. Cournot is a bridge: imperfect on one end, competitive on the other.

13 Problem 13 — Robinson Crusoe (Cumulative Synthesis)

Problem Statement

Robinson Crusoe lives alone on an island, and is simultaneously the only *firm* and the only *consumer*. He has utility over coconuts c and leisure ℓ: $U(c, \ell) = \ln c + \ln \ell$, with time endowment $T = 24$. Production: $c = \sqrt{L}$, where $L = T - \ell$ is labor.

(A) *Centralized planner.* Solve $\max U$ subject to the technology directly.

(B) *Decentralized economy.* Let w be the wage, p_c the coconut price (set $p_c = 1$). Solve the firm's problem (max profit), then Crusoe's consumer problem (taking w, receiving profit π).

(C) Show that the decentralized equilibrium yields the same allocation as the planner. State which welfare theorem this illustrates.

(D) Edge case: if the technology becomes $c = L$ (linear, CRS), what happens to the wage and profit?

Memory Anchor — "The Single-Brain Economy"

Robinson is a one-brain economy — planner and entrepreneur and consumer all in one skull. The *whole point* of Robinson Crusoe is to feel, viscerally, that prices and wages are *just bookkeeping* when there is one agent. The planner's solution and the market's solution *must* coincide — if they didn't, you'd see the same brain disagreeing with itself, which is impossible. The First Welfare Theorem is just "no schizophrenia."

Pattern Recognition Cue

Robinson Crusoe = Consumer + Producer + Equilibrium glued together. **Recipe: (1)** planner's problem (intuition); **(2)** firm's problem $\Rightarrow$ labor demand $L^d(w)$, profit $\pi(w)$; **(3)** consumer's problem with income $= wT + \pi$; **(4)** market clearing: $L^d(w) = T - \ell^*(w)$ pins down w^*.

Part (A) — Planner

Substitute $c = \sqrt{T - \ell}$:

$$\max_{\ell \in [0,T]} \tfrac{1}{2} \ln(T - \ell) + \ln \ell.$$

FOC: $\dfrac{-1/2}{T - \ell} + \dfrac{1}{\ell} = 0 \Rightarrow \ell = 2(T - \ell) \Rightarrow \ell^* = \frac{2T}{3} = 16$. Hence $L^* = T - \ell^* = 8$, $c^* = \sqrt{8} = 2\sqrt{2}$.

Part (B) — Decentralized Equilibrium

Firm: $\max \pi = \sqrt{L} - wL$. FOC: $\frac{1}{2\sqrt{L}} = w \Rightarrow L^d(w) = 1/(4w^2)$. Profit: $\pi(w) = \sqrt{1/(4w^2)} - w \cdot 1/(4w^2) = 1/(2w) - 1/(4w) = 1/(4w)$.

Consumer: income $m = wT + \pi(w) = 24w + 1/(4w)$. Cobb-Douglas in (c, ℓ) with weights $1/2, 1/2$ in the log $\Rightarrow$ spend half on each:

$$c^* = m/2, \qquad w\ell^* = m/2 \Rightarrow \ell^* = m/(2w).$$

Market clearing for labor: $L^d(w) = T - \ell^*$:

$$\frac{1}{4w^2} = 24 - \frac{24w + 1/(4w)}{2w} = 24 - 12 - \frac{1}{8w^2} = 12 - \frac{1}{8w^2}.$$

$$\frac{1}{4w^2} + \frac{1}{8w^2} = 12 \Rightarrow \frac{3}{8w^2} = 12 \Rightarrow w^{*2} = \frac{1}{32} \Rightarrow w^* = \frac{1}{4\sqrt{2}}.$$

Compute allocation: $L^* = 1/(4w^{*2}) = 1/(4 \cdot 1/32) = 8$. ✓ Matches planner. $\ell^* = T - L^* = 16$. ✓ $c^* = \sqrt{8} = 2\sqrt{2}$. ✓

Part (C) — First Welfare Theorem in Action

The decentralized allocation *exactly* replicates the planner's. This is the **First Welfare Theorem**: any competitive equilibrium is Pareto efficient. Prices are not "real" — they are a coordinating signal that lets a decentralized economy mimic a centralized optimizer. The brain has split into "firm" and "consumer," but they negotiate via w^* to land exactly where the unified brain would have.

Part (D) — The CRS Knife-Edge

With $c = L$: profit $= L - wL = (1 - w)L$. If $w < 1$, firm wants $L \to \infty$ (no equilibrium). If $w > 1$, firm shuts down. Equilibrium requires $w^* = 1$, profits $= 0$. **Free-entry zero-profit condition** — the same condition that pins long-run competitive prices in any CRS industry. (Recall the SOC discussion in Problem 6.)

Cross-Link — The Whole Course in One Problem

Robinson Crusoe touches every module: utility max (Module 1), Lagrangian / Cobb-Douglas demand (Problem 1), labor-leisure (Problem 4), profit max with SOC (Problem 6), conditional factor demand (Problem 5), market clearing and Walras' Law (Problem 9), and the welfare theorem. *If you can do this problem cold, you have the entire course.*

Part IV — Master Mnemonic Vault

14 The Pattern-Recognition Atlas

Trigger ⇒ Tool: One-Glance Lookup

When you see...	Reach for...
$U = x^a y^b$	Income-share rule, $x^* = \frac{a}{a+b}\frac{m}{p_x}$.
$U = \min(\cdot)$ (Leontief)	Equate the kink, sub into budget.
$U = v(x) + y$ (quasilinear)	No income effect; CS exact.
"Find Hicksian" or "expenditure"	Dual problem; Shephard's Lemma.
"Decompose total"	Slutsky equation.
"Show direction without solving"	Implicit Function Theorem.
"Show maximum" / 2-variable optimum	Hessian: signs of leading principals.
"Per-unit tax" or "ad valorem"	IFT on $D(p) = S(p - t)$; elasticity formula.
"Segmented markets" / discrimination	Inverse-elasticity per market.
"Risky payoff" / "insurance"	Expected utility, CE, Arrow-Pratt.
CARA + Normal payoff	$CE = E[w] - \frac{\gamma}{2}\text{Var}(w)$.
"Pollution" / "spillover"	Pigouvian tax = marginal external damage at q^S.
"Two firms" / "best response"	Cournot FOC, symmetric solution.
Consumer + firm together	Robinson Crusoe; First Welfare Theorem.
"Pure exchange" / Edgeworth	Walras' Law; tangent of indifference curves.

15 The Twelve Anchors (Memorize as One Sentence Each)

The Anchor Stack

1. **Fixed-Share Wallet** (Cobb-Douglas): pockets stitched in fixed proportions.
2. **Necessity Bucket** (quasilinear): a bucket with a lid; rain runs off into the pool.
3. **Patience Tug-of-War** (intertemporal Euler): two hands on a rope, βR vs 1.
4. **Rubber Band** (labor supply): leisure snaps back; wages magnetize.
5. **Mirror Dimension** (cost minimization): producer theory is consumer theory reflected.
6. **Bowl Holding Water** (profit-max SOC): NSD Hessian = bowl-down = unique max.
7. **Inelastic Sucker** (tax incidence): stiffer rope absorbs the tug.
8. **Bicycle Lock** (price discrimination): independent dials per market.
9. **Auctioneer's Coin** (general equilibrium): one relative price clears all markets.
10. **Concave Hill** (risk aversion): chord above curve; gap = risk premium.
11. **Phantom Cost** (externality): Pigou makes the invisible visible.

12. **Two Wolves** (Cournot): each takes the other's bite as fixed.
13. **Single-Brain Economy** (Robinson Crusoe): no-schizophrenia = welfare theorem.

16 Final-Week Spaced-Retrieval Schedule

A Three-Day Consolidation Plan

Day −3: Read all Memory Anchors. Cover solutions; attempt Problems 1, 5, 9, 10, 13 (one per part) cold. Allow yourself the formula sheet *only* after a 10-minute attempt.
Day −2: Re-attempt the problems you missed yesterday. Add Problems 3, 7, 11 (the IFT/comparative statics trio). Rehearse the Pattern Recognition Atlas as a flashcard exercise — 3 passes.
Day −1: Targeted review. Open the failure list from Day −2; redo each. Then *teach* Robinson Crusoe (Problem 13) out loud as if to another student. Teaching is the densest form of retrieval practice known — it forces every gap to surface.
Exam morning: Look only at the Pattern Recognition Atlas & the Twelve Anchors. Do not re-derive. Re-derivation under time pressure burns glucose; pattern matching does not.

"Mathematics is the language with which God has written the universe." — Galileo
Microeconomics is that language applied to human decisions.